Gaussian Radial-Basis-Function solution of the non-linear Fokker-Planck equation

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The non-linear Fokker-Planck collision operator [1] that describes the collisional evolution of a plasma involves velocity-space friction and diffusion coefficients that are weighted integrals of the particle distribution function. Unfortunately, for a non-Maxwellian distribution function, these coefficients are difficult to compute. In this contribution, we present a completely new idea to address this difficulty and to solve the non-linear Fokker-Planck equation.

Observing that the friction and diffusion coefficients have analytical expressions for shifted Gaussian distributions motivates a Gaussian *Radial-Basis-Function* (RBF) approach – a method that is often encountered in interpolation of scattered multidimensional data. We express the distribution function as a sum of Gaussian RBFs and derive an exact expression for the full nonlinear collision operator. Further, using the RBF method, we demonstrate the solution of a non-linear relaxation problem both in the axisymmetric case, and in the full 3-D velocity space. The quality and conservative properties of the solution are also discussed.

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[1] M. N. Rosenbluth, W. M. MacDonald and D. L. Judd, *Phys. Rev.* **107**, 1 (1957).

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