Profile Evolution and Momentum Transport in the Core and Pedestal

Peter J. Catto

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Special thanks to Felix Parra, Michael Barnes, Jungpyo Lee

EFTC Lisbon 5-8 October 2015

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Questions

- Why do we have to be careful evaluating core momentum transport and evolving profiles?
- Can we evaluate core intrinsic rotation?
- What changes in the pedestal?

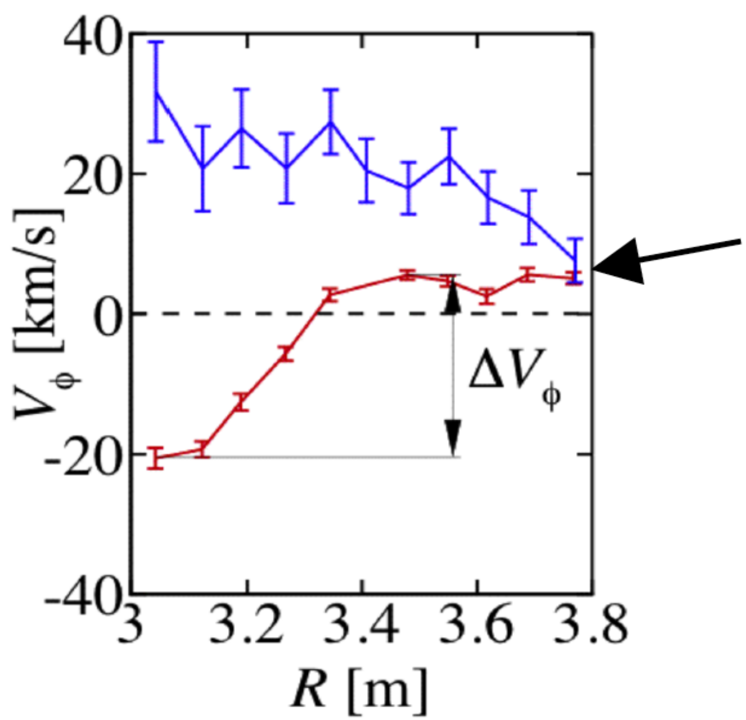
Perspective

- Decidedly neoclassical: "legendary figures" of plasma theory did not try to directly evaluate collisional momentum transport
- To evolve ion flow $\vec{V}$ need to find $\vec{E} = -\nabla\Phi$ by evaluating momentum transport since
$$\vec{V} = cB^{-1}\vec{b} \times [\nabla\Phi + (Zen)^{-1}\nabla p] + V_{\|}\vec{b}$$
Flux function portion of Φ harder to evaluate than n & T. $V_{\|}$ is evaluated kinetically
- Tricks of "legends" work with turbulence!

Motivation: Intrinsic rotation

- Rotation beneficial for MHD and turbulence
- Intrinsic rotation = momentum redistribution with little or no momentum input
- Mostly intrinsic rotation in ITER & reactors
- Intrinsic rotation results in diverse behavior

Two different JET ICRH shots



Same rotation at the edge

To explain the different behaviors we need to understand momentum transport and profile evolution

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Difficult: a small error in ambipolarity leads to an unphysical torque!

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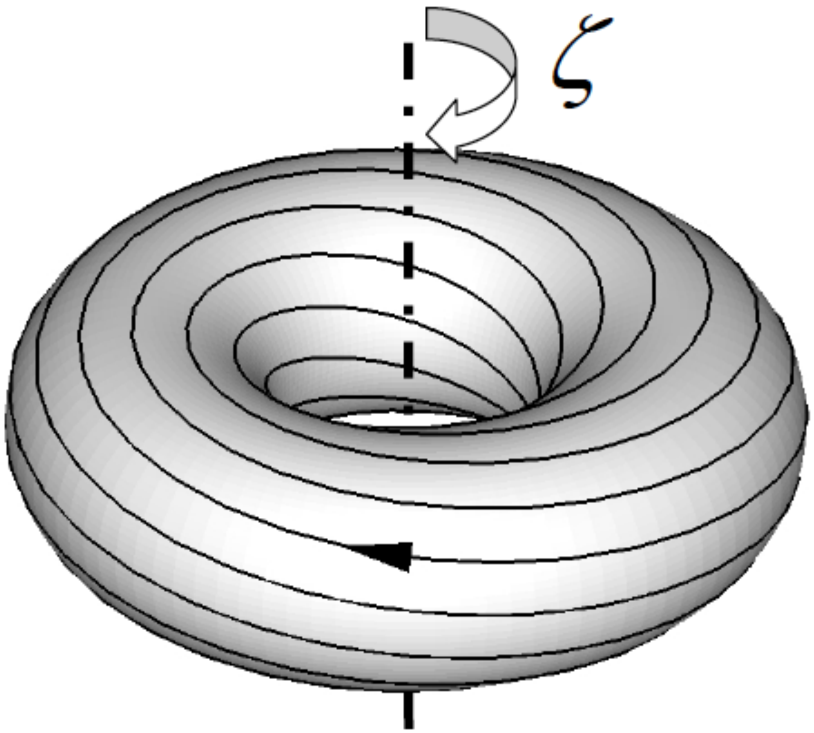
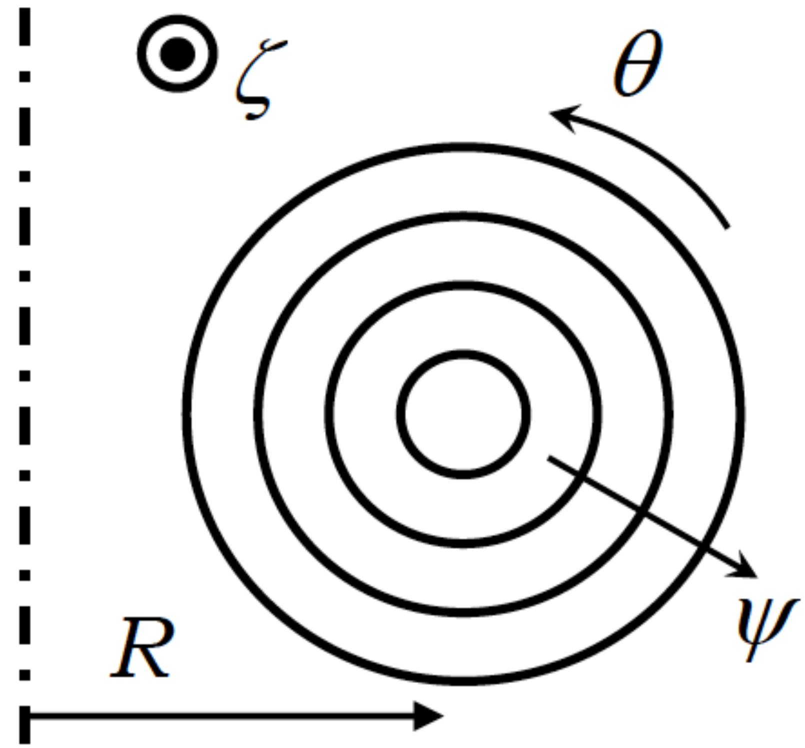
Errors:

- Analytic: gyrokinetic equation is not exact
- Numerical: due to noise and/or algorithms

Axisymmetric geometry



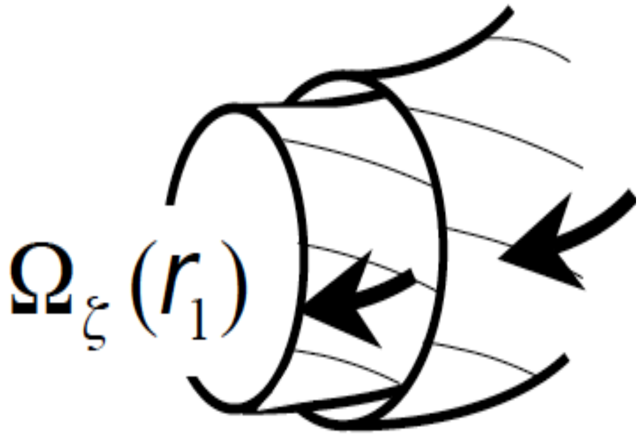


- $\vec{B} = I(\psi)\nabla\zeta + \nabla\zeta \times \nabla\psi$, $\nabla\zeta$ co-current direction
- Unperturbed ion flow $\vec{V} = \Omega_\zeta R^2 \nabla\zeta + U(\psi)\vec{B}$

  $U \propto \partial T_i/\partial\psi$ & if sonic $\Omega_\zeta \Rightarrow -c\partial\Phi/\partial\psi$

Angular momentum cons. determines E_{radial}

$$\left\langle \frac{RB_p}{c} J_r \right\rangle - \frac{\partial}{\partial t}\langle MnRV_\zeta \rangle = \langle \nabla\cdot(R^2 \overleftrightarrow{\pi}\cdot\nabla\zeta)\rangle \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}(rR\pi_{r\zeta})$$



$$\Omega_\zeta(r_2) = \frac{c}{RB_{pol}}\left(E_r - \frac{1}{en}\frac{\partial p_i}{\partial r}\right)$$

differential rotation

$\pi_{r\zeta} = \pi_{\zeta r}$ off diagonal stress tensor

$\langle \ldots \rangle$ = flux surface average

- Ambipolarity error $\langle RB_p J_r \rangle \neq 0 \Rightarrow$ a torque!

Scale separation

- Turbulence $\Rightarrow$ eddy size = Δ << a = global size

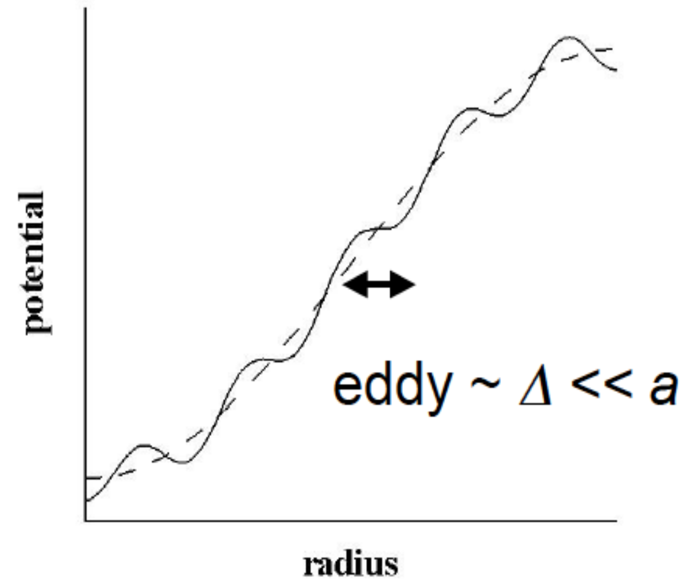
$f = F + \delta f$ with $\delta f \sim Fe\delta\Phi/T$

$\nabla F \sim F/a \sim \delta f/\Delta \sim \nabla \delta f$

Evolution: $\partial F/\partial t \sim D_{turb} F/a^2$

$\delta F \sim F\rho_p/a$ in $F = f_{Max} + \delta F$

$\rho_p = \rho B/B_p$



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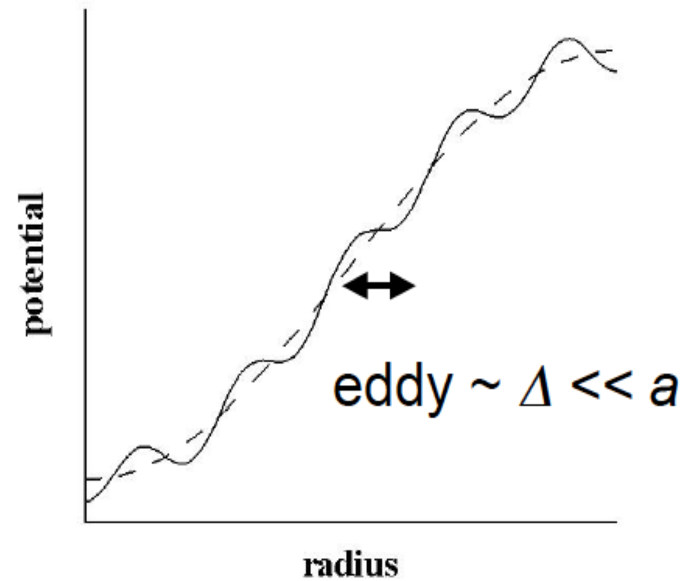

- Anisotropic fluctuations along & across $\vec{B}$

$$\nabla_{||}\delta f \sim \delta f/qR$$

qR = connection length with $B_p/B \sim a/qR << 1$

- Many eddy turn over times to cross core

Diffusivity estimate from GK critical balance

(critical balance ⇒ Barnes, Parra & Schekochikin PRL 2011)

- Nonlinear $\delta\vec{E}\times\vec{B}$ ~ gradient drive

$$\delta\vec{V}_E\cdot\nabla_\perp\delta f \sim \delta\vec{V}_E\cdot\nabla F \;\Rightarrow\; \delta f/F \sim e\delta\Phi/T \sim \Delta/a$$

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- Eddy size = $\Delta \sim k_\perp^{-1}$: note $\delta\vec{V}_E \sim \rho v_i/a$

$$v_\parallel\nabla_\parallel\delta f \sim \delta\vec{V}_E\cdot\nabla_\perp\delta f \;\Rightarrow\; \Delta \sim \rho qR/a \sim \rho_p$$

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- Eddy turnover time = τ with $v_i = (2T/M)^{1/2}$

$$v_\parallel\nabla_\parallel\delta f \sim \delta f/\tau \;\Rightarrow\; \tau \sim qR/v_i \sim 1/\omega_*$$

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- Turbulent diffusivity = $D_{turb} \sim \Delta^2/\tau$

$$D_{turb} \sim \rho_p^2 v_i/qR \sim (qR/a)D_{gB} >> D_{plateau} > D_{banana}$$

(consistent with Table 4 of Parra & Barnes PPCF 2015)

Ambipolarity error due to $\langle \nabla \cdot \vec{J} \rangle \neq 0$

- To avoid ambipolarity error from $\langle RB_p J_r \rangle \neq 0$

$$\partial \pi_{r\zeta}/\partial r >> c^{-1} B_p J_r^{error} \Rightarrow J_r^{error}/env_i << \pi_{r\zeta} \rho_p/nTa$$

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- Assume momentum diffuses: $\pi_{r\zeta} \sim D_{turb} \nabla(Mn\vec{V})$

$D_{turb} \sim \rho_p^2 v_i/qR$ & diamagnetic flow $\vec{V} \sim \rho_p v_i/a$

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- Direct evaluation: $\pi_{r\zeta}/nT \sim (\rho_p/a)^2(\rho/a) \sim \delta f/f_{Max}$

Standard gyrokinetic equation: $\vec{R} = \vec{r} + \Omega^{-1}\vec{v}\times\vec{b}$

- In $E=v^2/2+Ze\Phi/M$, $\mu = v_\perp^2/2B$ velocity variables, lowest order gyrokinetic eq. for $f = F + \delta f$

$$\frac{\partial f}{\partial t} + \frac{d\vec{R}}{dt}\cdot[\nabla_R f - \frac{Ze}{M}\nabla_R\langle\Phi\rangle_R \frac{\partial f}{\partial E}] = \langle C\{f\}\rangle_R$$

$$\frac{d\vec{R}}{dt} = v_\parallel\vec{b} - \frac{c}{B}\nabla_R\langle\Phi\rangle_R\times\vec{b} + \vec{v}_{Magnetic}$$

$\langle\ldots\rangle_R$ = gyrophase average at fixed $\vec{R}$, & $\vec{b} = \vec{B}/B$

- $\delta f/F \sim \rho/a$ & retains $k_\perp\rho \sim 1$: $(\rho_p/a)^2$ missing!

- OK for heat and particle fluxes using moments

Intrinsic ambipolarity

- Intrinsic ambipolarity means $\langle \nabla \cdot \vec{J} \rangle = 0$ or $\langle \vec{J} \cdot \nabla \psi \rangle = 0$ independent of radial $\vec{E} = -\nabla \Phi$
- Stellarators are not intrinsically ambipolar unless they are quasi-symmetric (omnigeneity is not enough) so $\langle \vec{J} \cdot \nabla \psi \rangle = 0$ gives $\partial \Phi / \partial \psi$

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- Tokamaks intrinsically ambipolar to order $\delta f/F \sim (\rho/a)^2 \Rightarrow$ next order GKE not enough for a direct evaluation of $\pi_{r\zeta}$
  (see Parra & Catto PPCF 2009 and Sugama *et al.* PPCF 2011)

ugh!

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- Seems hopeless!

But there is an implementable method to evaluate core intrinsic rotation and it works!

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Moment approach: origin

$$\Omega \vec{v} \times \vec{b} \cdot \nabla_v f >> \frac{\partial f}{\partial t}, v_{\parallel} \vec{b} \cdot \nabla f, C\{f\}$$

$$(\Omega >> \omega_*, v_i/qR, \nu)$$

Moment approach to the rescue!

- Only requires $\delta f/f_{Max} \sim (\rho_p/a)(\rho/a)$
- Moment approach: use velocity moments of FP for $\rho_p << a$ to evaluate tor. ang. mom. flux

$$\Pi \equiv M\langle R^2 \int d^3v\, f\, \nabla\zeta \cdot \vec{v}\vec{v} \cdot \nabla\psi \rangle_T \approx R^2 B_p \pi_{r\zeta} \text{ in}$$

$$\frac{\partial}{\partial t}\langle MnR^2\vec{V}\cdot\nabla\zeta\rangle_T + \frac{1}{V'}\frac{\partial}{\partial\psi}(V'\Pi) = \text{applied torque}$$

$$\langle \ldots \rangle_T = (\Delta t \Delta\psi)^{-1}\int_{\Delta t} dt \int_{\Delta\psi} d\psi \langle \ldots \rangle \text{ with}$$

$$\langle \ldots \rangle = (V')^{-1}\oint d\vartheta d\zeta(\ldots)/\vec{B}\cdot\nabla\vartheta \;\&\; V' = \oint d\vartheta d\zeta/\vec{B}\cdot\nabla\vartheta$$

Direct moment approach: particle flux example

- Complex so illustrate using particle transport

$$\frac{\partial n}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\langle n\vec{V}\cdot\nabla\psi\rangle_T\right) = 0$$

- Direct $\langle n\vec{V}\cdot\nabla\psi\rangle_T$ evaluation requires:

$$\langle \delta n \delta\vec{V}\rangle_T\cdot\nabla\psi \sim (n\rho_p/a)(v_i\rho/a)RB_p,$$

$$\langle \delta n\rangle_T\vec{V}\cdot\nabla\psi \quad \text{and} \quad n\langle\delta\vec{V}\rangle_T\cdot\nabla\psi$$

- $\langle \delta n\rangle_T$ & $\langle\delta\vec{V}\rangle_T$ only vanish to lowest order

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- $\vec{V}\cdot\nabla\psi=0$ & $\vec{V}\sim v_i\rho_p/a\Rightarrow$ do not need $\langle\delta n\rangle_T$
- Standard GKs doesn't give $\delta\vec{V}$ to order $v_i\rho\rho_p/a^2$
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- Standard GKs doesn't give $\delta\vec{V}$ to order $v_i\rho\rho_p/a^2$
- Gain an order in ρ_p/a using a moment of FP eq.

Indirect moment approach: particle flux

- Momentum conservation gives $\langle n\vec{V}\cdot\nabla\psi\rangle_T =$

$$\langle\frac{\nabla\psi\times\vec{b}}{\Omega}\cdot[\frac{\partial}{\partial t}(n\vec{V})+\nabla\cdot(\int d^3vf\vec{v}\vec{v})+\frac{Zen}{M}\nabla\Phi-\int d^3v\vec{v}C]\rangle_T$$

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- Using $\nabla\psi\times\vec{b}=R^2B\nabla\zeta-I\vec{b}$ then parallel mom. (turbulent) + (class. + neo. cl. collisional)

$$\langle n\vec{V}\cdot\nabla\psi\rangle_T = c\langle n\partial\Phi/\partial\zeta\rangle_T-(Mc/Ze)\langle R^2\int d^3v\,\vec{v}\cdot\nabla\zeta C]\rangle_T$$

Remaining terms small by ρ_p/a or less, and

$$R^{-1}B_p^{-1}\langle cn\partial\Phi/\partial\zeta\rangle_T\Rightarrow\langle\delta n\delta\vec{V}_E\rangle_T\sim D_{turb}n/a$$

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- Gain an order in ρ_p/a as in neoclassical theory

Moment approach: heat & momentum fluxes

- To evaluate ion heat transport need

$$\left\langle \int d^3v f v^2 \vec{v} \cdot \nabla\psi \right\rangle_T \quad \text{and} \quad \left\langle n\vec{V} \cdot \nabla\Phi \right\rangle_T$$

  Evaluate $\left\langle n\vec{V} \cdot \nabla\Phi \right\rangle_T$ as for particle flux

  Use $\vec{v}v^2$ FP moment

  Starts getting complicated!

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- For momentum transport need $\vec{v}\vec{v}\vec{v}$ moments

(Parra & Catto PPCF 2010, Parra *et al.* NF 2012, Parra & Barnes PPCF 2015)

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- $\Pi \equiv M \langle R^2 \int d^3v f \nabla\zeta \cdot \vec{v}\vec{v} \cdot \nabla\psi \rangle_T \Rightarrow$ many terms! turbulent, neoclassical, & finite orbit width + combinations; radial derivative + slow poloidal fluctuation variations; up-down asymmetry;...
- Momentum transport needs nonlocal features

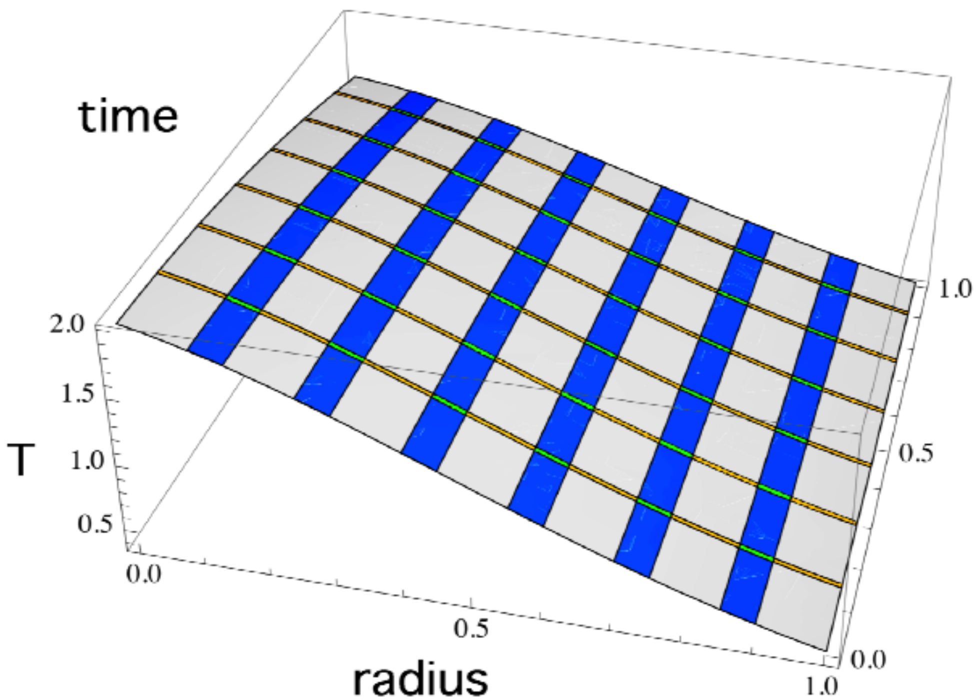
Next order gyrokinetic equation not enough!

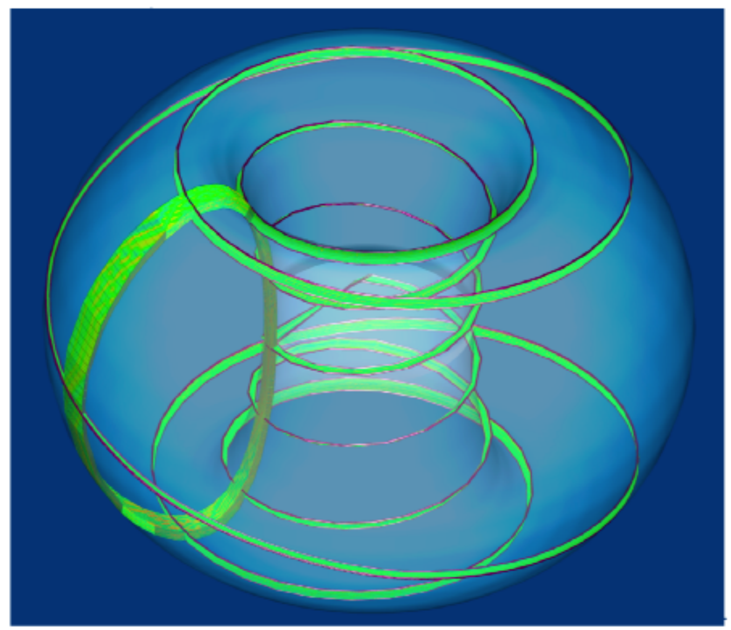
- Cannot solve ρ_p/a corrected GKE to evolve profiles directly: need $\delta f/f_{Max} \sim (\rho_p/a)^2(\rho/a)$ and it only gives $\delta f/f_{Max} \sim (\rho_p/a)(\rho/a)$

- Coupling ρ_p/a corrected GKE to a fluid code evolving n, T and Φ picks up another power of ρ_p/a with moment approach for Π

Hybrid gyrokinetic + fluid & multi-scale ($\Delta \ll a$)

- GS2 with higher order GKE plus Trinity with conservation eqs. treats momentum transport and evolves profiles
- Turbulent GS2 fluctuations on fine space-time grid embedded in coarse TRINITY "fluid" grid





*Barnes et al., Phys. Plasmas (2010)

Toroidal angular momentum conservation

$$\frac{1}{r}\frac{\partial}{\partial r}(rR\pi_{r\zeta}) = 0$$

- Steady state with no applied torque: $\pi_{r\zeta}(r=0)$

$$\pi_{r\zeta}(r) = 0$$

- Find Ω_ζ or $\langle\Phi(r)\rangle$ by solving $\pi_{r\zeta}(r) = 0$
- Limits: high flow-sonic & low flow-diamagnetic
- ITER diamagnetic, but sonic limit of some interest: no intrinsic or residual stress

Strong rotation or sonic limit

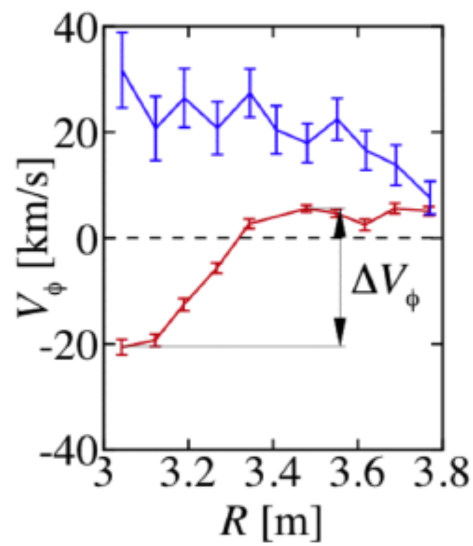
- Expand $\pi_{r\zeta}(\Omega_\zeta, \partial\Omega_\zeta/\partial r)$ for small Ω_ζ & $\partial\Omega_\zeta/\partial r$:

$$\pi_{r\zeta}/MnR = -P\Omega_\zeta - D\partial\Omega_\zeta/\partial r$$

- "Intrinsic" rotation means no source ⇒ pinch P + diffusion D depend on n, T, $\partial n/\partial r$ & $\partial T/\partial r$

$$P\Omega_\zeta + D\partial\Omega_\zeta/\partial r = 0 \Rightarrow \Omega_\zeta = \Omega_\zeta(a)\exp\left(\int_r^a dr P/D\right)$$

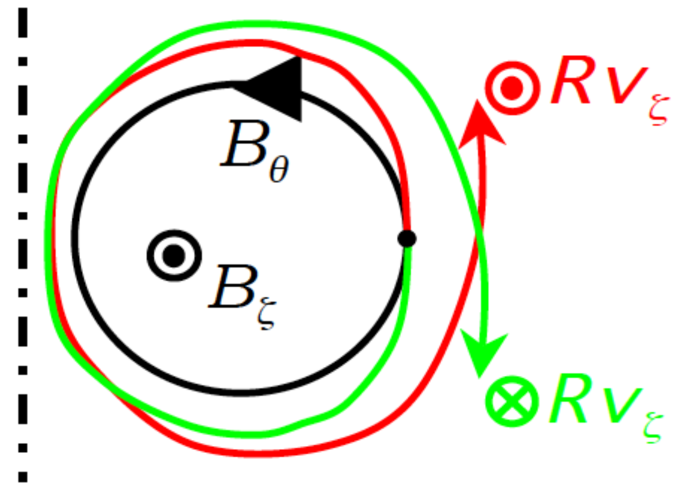
- No sign change! Red curve?
- Sign depends on edge
- Important symmetries ⇒ symmetry breaking in Π_{int}



Parra et al., PRL (2012)

High flow symmetry properties

- Up-down symmetric
- Changing signs of Ω_ζ, $\partial\Omega_\zeta/\partial\psi$, ϑ, $v_{||}$, k_ψ changes signs of δf and $\delta\Phi$
  (see Parra, Barnes & Peeters 2011)
- $\vec{V} \sim v_i$ easier by ρ_p/a



no ang. mom. change
due to ions of opposite $v_{||}$

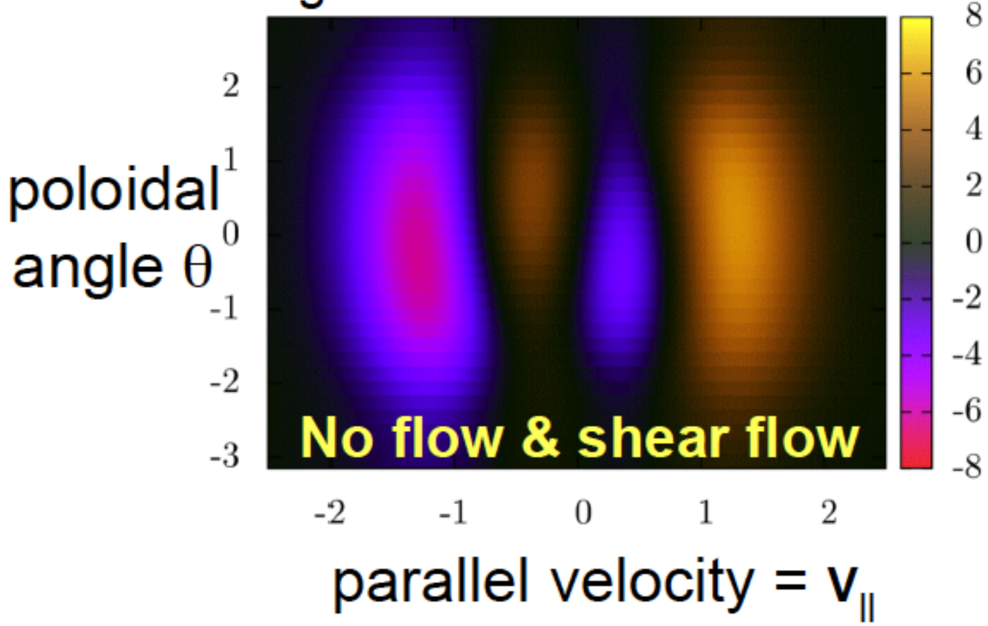
- $\Pi(\Omega_\zeta=0, \partial\Omega_\zeta/\partial\psi=0) = 0$
- Expanding gives $\Pi \propto -P\Omega_\zeta - D\partial\Omega_\zeta/\partial r$
- Sign change of Ω_ζ & $\partial\Omega_\zeta/\partial\psi$ changes sign of Π

Up-down high flow symmetry

Radial angular momentum flux: integrand odd when

$$\Omega_\zeta = 0 = \partial\Omega_\zeta/\partial r$$

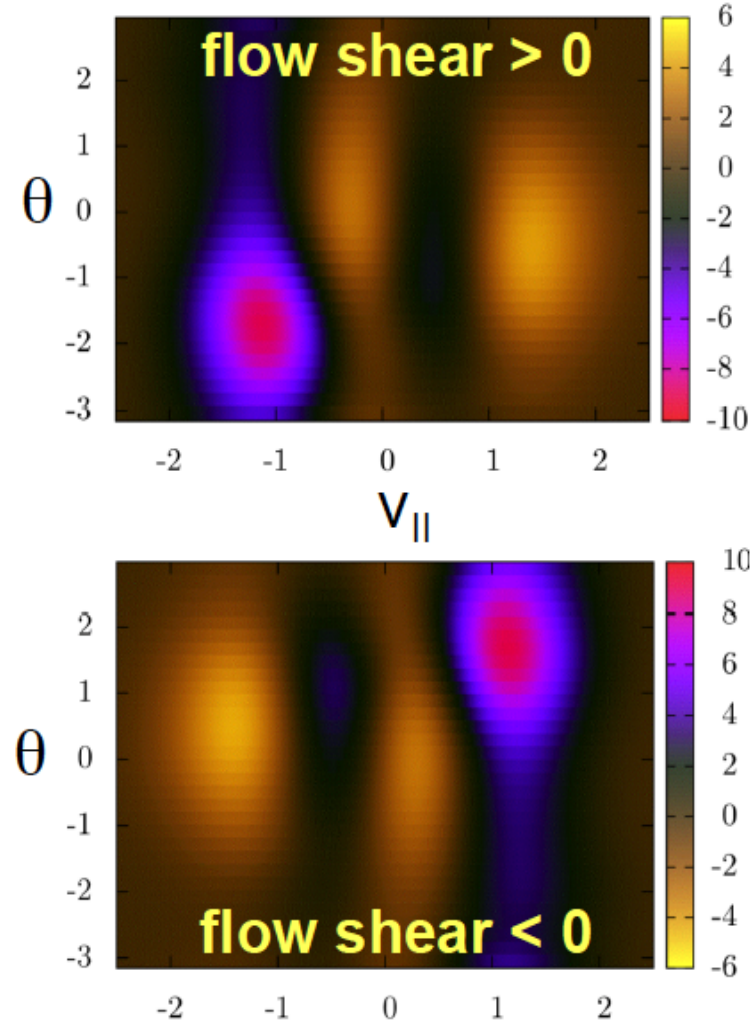
Integrand contribution to flux = 0



$V_\zeta \sim v_i$ & $\delta f/F \sim \rho_p\rho/a^2$

$$\partial\Omega_\zeta/\partial r \neq 0 = \Omega_\zeta$$

Net integrand contribution to flux



Diamagnetic flow: up-down symmetric tokamak

- Symmetry: $\Pi \equiv M\left\langle R^2 \delta\vec{V}_E \cdot \nabla\psi \int d^3v\, \delta f\, \vec{v} \cdot \nabla\zeta \right\rangle_T \approx 0$

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- Next order, phenomenological form popular

$$\pi_{r\zeta}/MnR = -P\Omega_\zeta - D\partial\Omega_\zeta/\partial r + \pi_{int}/MnR,$$

with $Pa \sim D$ & intrinsic or residual stress $= \pi_{int}$

$$\pi_{int}/MnR \sim (v_i\rho_p)^2\rho/a^3R$$

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- Use $\partial\Omega_\zeta/\partial r \sim \Omega_\zeta/a$ & recall $D_{turb} \sim \rho_p^2 v_i/qR$

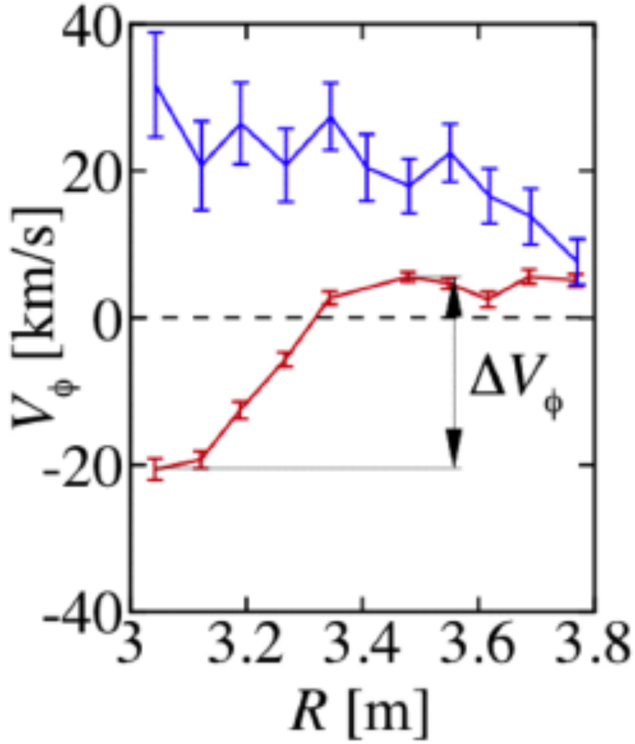
  For intrinsic rotation expect $\pi_{r\zeta} = 0$ to give

$$R\Omega_\zeta \sim v_i\rho_p/a$$

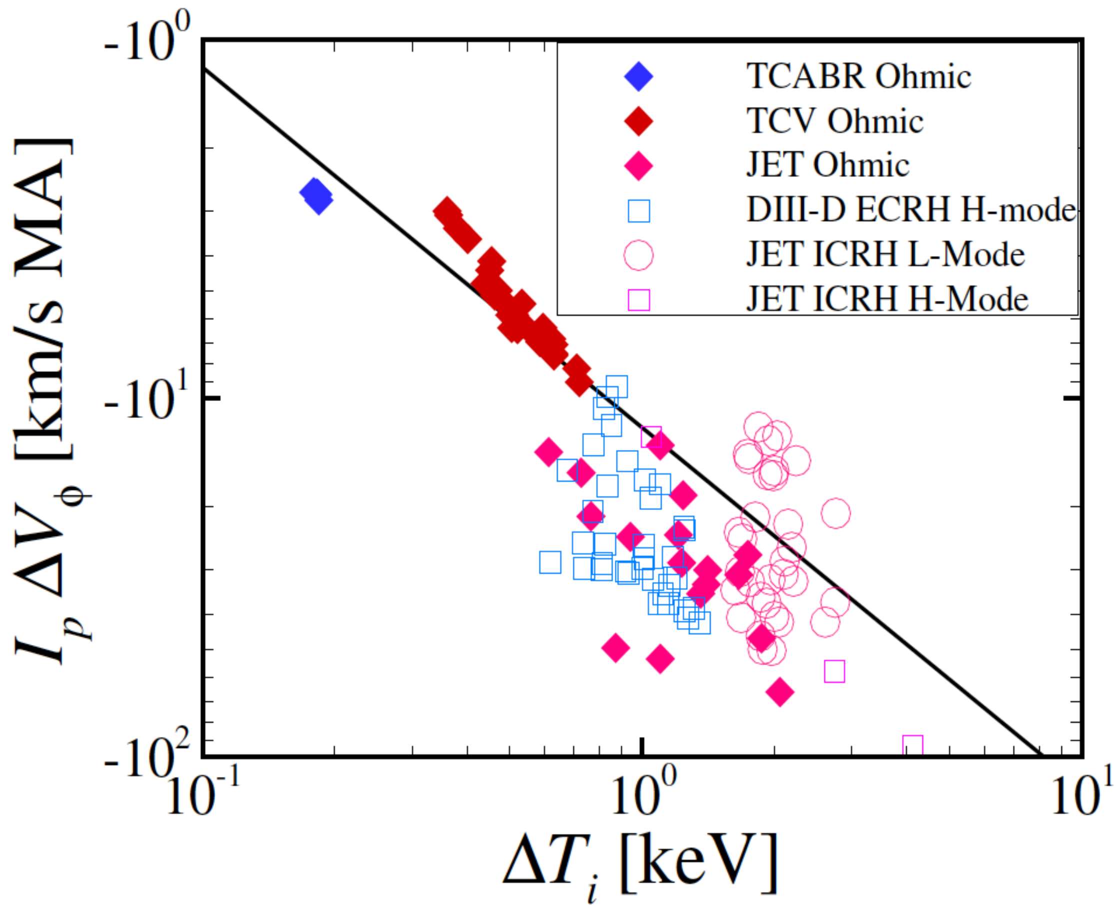
- π_{int} matters - essential when rotation changes sign $\Rightarrow$ hollow profiles

Diamagnetic ion flow ordering: hollow profiles

$$\Pi = 0 \;\; \Rightarrow \; V_\phi = V_\zeta = R\Omega_\zeta \sim v_i\rho_p/a$$



Parra et al., PRL (2012)



$$2\pi B_p a \approx 4\pi c^{-1} I_p \;\; \Rightarrow V_\zeta = R\Omega_\zeta \sim c^2 T/eI_p$$

Diamagnetic flow ordering

- Recalling $\Omega_\zeta = \Omega_E + \Omega_d$, find $\Pi_{int} \Rightarrow$ 2 pinches and 2 diffusivitiies when $\vec{V} \sim \rho_p v_i / a$

$$\Pi = -Mn\langle R^2\rangle [P_E \Omega_E + P_d \Omega_d + D_E \frac{\partial \Omega_E}{\partial r} + D_d \frac{\partial \Omega_d}{\partial r}] + \Pi'_{int}$$

Π'_{int} = remaining intrinsic or residual stress

Diamagnetic flow ordering

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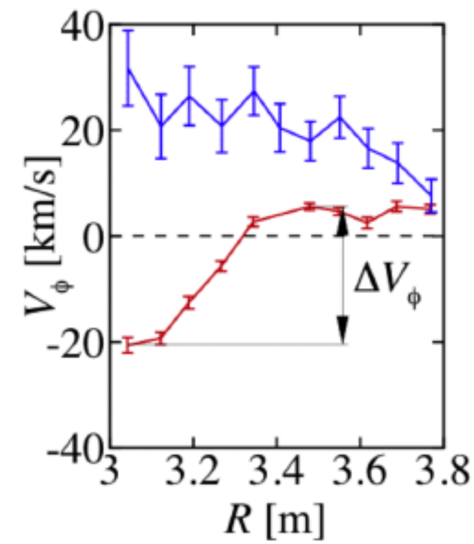
If $\Omega_d > 0$ at $\Omega_\zeta = 0$ & $D_d = D_E$:

Blue $\Rightarrow P_d > P_E$ mom. flux in =

$Mn\langle R^2\rangle(P_E - P_d)\Omega_d < 0 \Rightarrow$ peaked



Parra et al., PRL (2012)

Diamagnetic flow ordering

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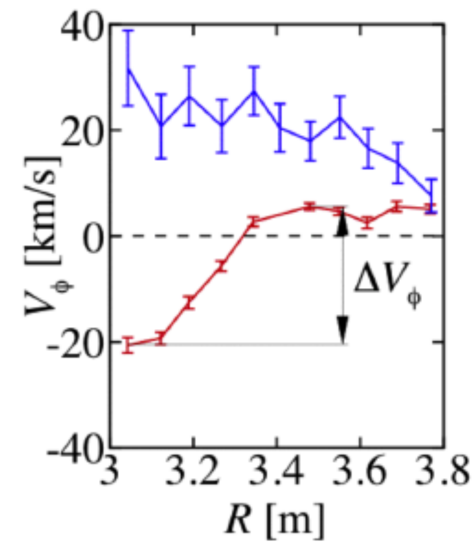
If Π'_{int} = remaining intrinsic or residual stress $\rightarrow 0$

& $\Omega_d > 0$ at $\Omega_\zeta = 0$ & $D_d = D_E$:

Blue $\Rightarrow P_d > P_E$ mom. flux in = $Mn\langle R^2\rangle(P_E - P_d)\Omega_d < 0 \Rightarrow$ peaked

Red $\Rightarrow P_d < P_E$ mom. flux out = $Mn\langle R^2\rangle(P_E - P_d)\Omega_d > 0 \Rightarrow$ hollow

(based on Lee, Parra & Barnes 2014)



Parra et al., PRL (2012)

What changes in the pedestal?

$\Downarrow$

Standard gyrokinetics requires care

$\Downarrow$

$$(v_{\parallel}\vec{b} - \frac{c}{B}\nabla_R \langle \Phi \rangle_R \times \vec{b}) \cdot \nabla_R F = \langle C\{F\} \rangle_R$$

Pedestal changes

- Pedestal adjacent to SOL
- $B_p/B \sim a/qR$ with $q >> 1$
- Pedestal width $\sim \rho_p \Rightarrow \partial f/\partial r \sim f/\rho_p \Rightarrow$ can be non-Maxwellian

- $E \times B$ and diamagnetic flow terms each sonic, but in opposite directions

$$cR\partial\Phi/\partial\psi \sim (cRT/Zen)\partial n/\partial\psi \sim v_i$$

- Unperturbed $E \times B$ and streaming compete

$$v_{\parallel} \sim cI\,B^{-1}\partial\Phi/\partial\psi \sim v_i$$

- Strong poloidal variation of n, T & Φ possible

Pedestal width $\sim \rho_p$

- Core: transit average $v_{\|}\vec{b}\cdot\nabla F = C\{F\} \Rightarrow f_{Max}$
- In pedestal can derive the GKE using ψ_*

$$\psi_* = \psi + \Omega^{-1}\vec{v}\times\vec{b}\cdot\nabla\psi - Iv_{\|}/\Omega$$

gyro $\Rightarrow \rho$ + drift $\Rightarrow \rho_p$

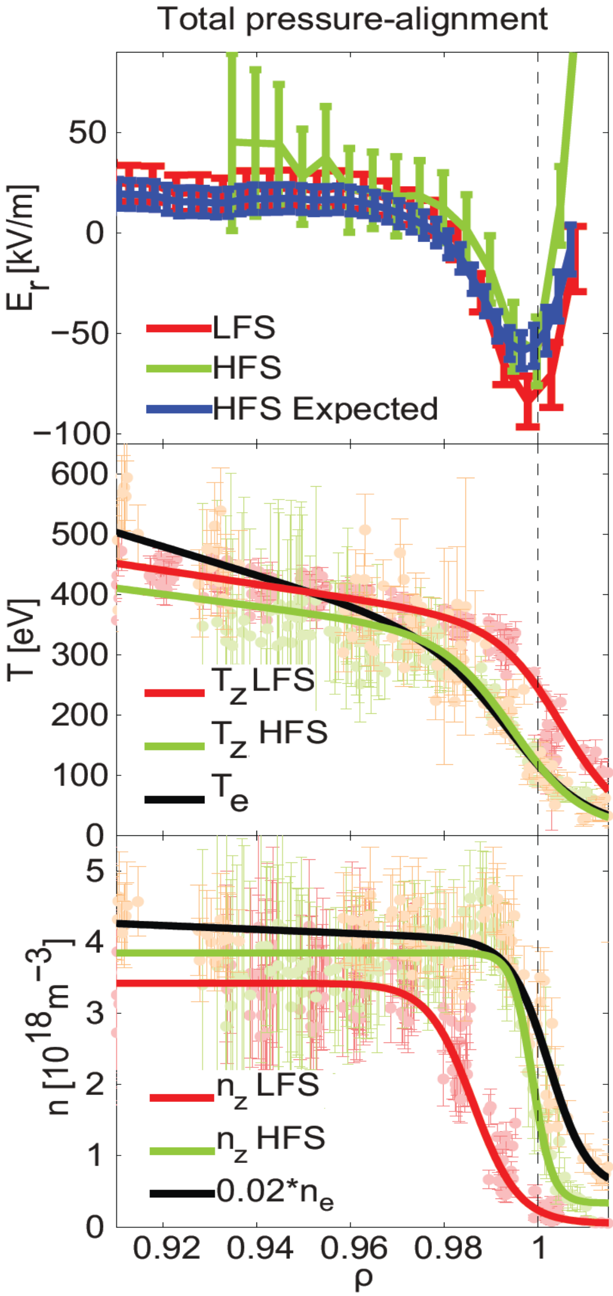
Gyroaverages at fixed ψ_* (Kagan & Catto 2008) but

$$f(\psi_*,E) - f(\psi,E) \sim \rho_p \partial f/\partial r \sim f$$

- Pedestal: finite orbit transit ave. @ fixed ψ_*

$$(v_{\|} + cIB^{-1}\partial\Phi/\partial\psi)\vec{b}\cdot\nabla F = C\{F\} \Rightarrow F = F(\psi,\vartheta,E,\mu)$$

Strong poloidal variation

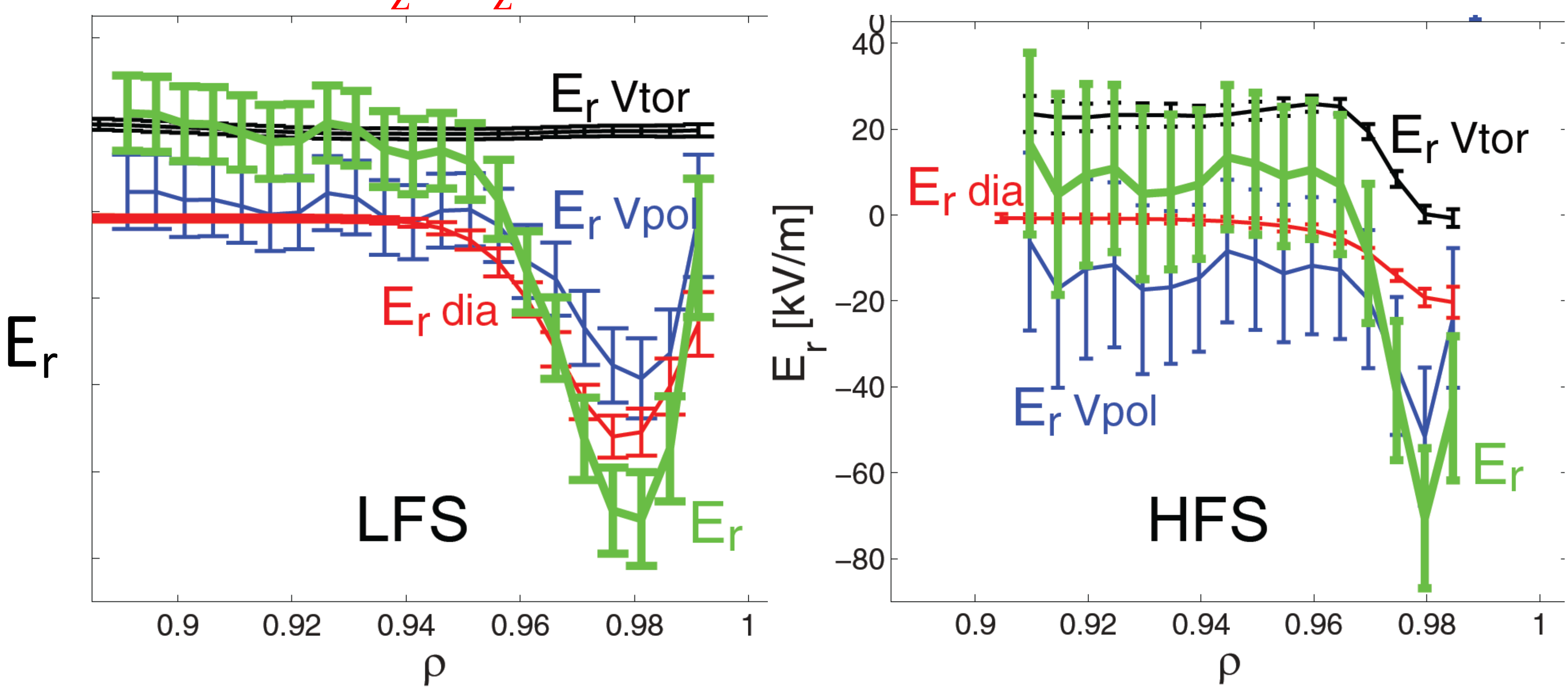


CX recombination spectroscopy on C-Mod observes poloidal variation of Φ & impurity n & T (Theiler *et al*. NF 2014 & Churchill *et al*. PoP 2015)

Stronger poloidal variation than B: must allow $\rho_p \sim a$ & sonic impurity flow, cannot neglect impurity diamagnetic drift, & impurity T not a flux function

Need impurity diamagnetic term

- Use $E_r = \dfrac{\partial p_z/\partial r}{Z_z e n_z} + B_{pol} V_{ztor} - B_{tor} V_{zpol}$ to measure



(Theiler *et al*. NF 2014)

Poloidal variation of E_r & diamagnetic term!

Strong poloidal variation with sonic flows

- Keep diamagnetic terms for ions & impurities

$$\frac{Z_i T_z}{Z_z n_z}\frac{\partial n_z}{\partial \psi} \sim \frac{T_i}{n_i}\frac{\partial n_i}{\partial \psi} \sim Z_i e \frac{\partial \Phi}{\partial \psi} \sim \frac{\partial T_i}{\partial \psi}$$

- E×B & dia. can't balance poloidally for both
- Flows vary poloidally from sonic to sub-sonic
- Alter pedestal model of Helander 1998 PoP:

$$\text{inertial} = M_z n_z \vec{V}_z \cdot \nabla V_{z\|} = R_{zi\|} = \text{friction}$$

$$\text{compress. heat.} = \frac{n_z}{T_z^{1/2}} \vec{V}_z \cdot \nabla\left(\frac{T_z^{3/2}}{n_z}\right) = Q_{zi} = \text{equilib.}$$

- Need $\vec{V}_z$ for poloidally varying n_z, T_z and Φ

(Espinosa & Catto EPS 2015)

Summary

- Practical way to handle core momentum transport & profile evolution
- Pedestal presents new challenges
- Related talk on up-down asymmetry today at 11am by J. Ball
- Related experimental results in a Tuesday poster by F. Parra