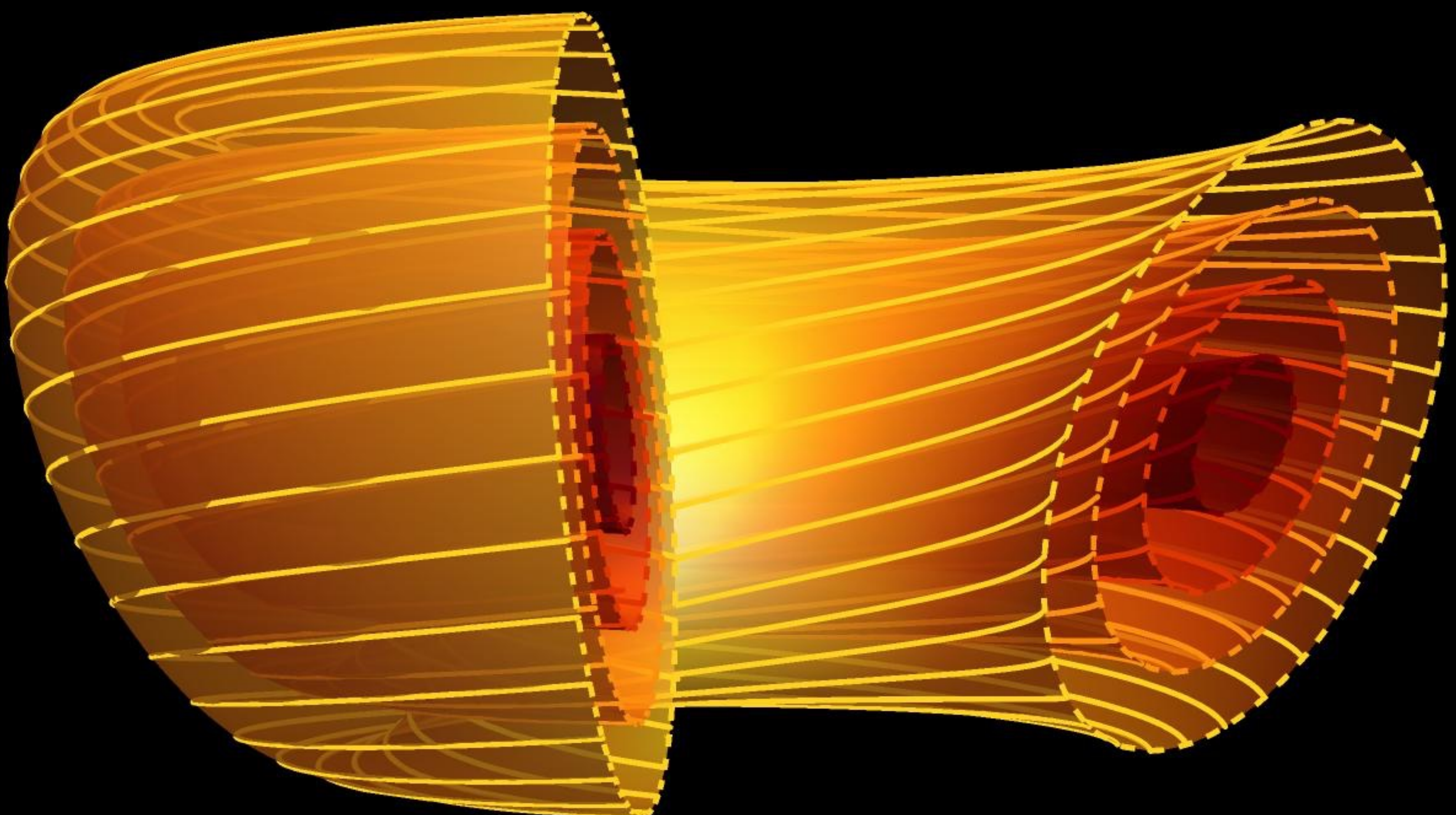
Intrinsic momentum transport in tokamaks with tilted elliptical flux surfaces



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Oxford University and CCFE

European Fusion Theory Conference
5 October 2015

The problem

Troyon et al. *PPCF* (1984).
Liu et al. *Nucl. Fusion* (2004).
Garofalo et al. *PRL* (2002).

- Make tokamaks work better
- Do this by increasing the limit on β: $\beta_N \equiv \frac{aB_0}{I_p}\frac{2\mu_0 \langle p \rangle}{B_0^2} \lesssim \beta_N^{\mathrm{Troyon}}$
- Nearly all reactor-scale devices excite instabilities by violating the β limit, e.g.

$$\beta_N \approx 2.57 > \beta_N^{\mathrm{Troyon}} \approx 2.45 \quad \text{(ITER)}$$

- Stabilizing ITER at $\beta_N = 3$ through toroidal rotation requires

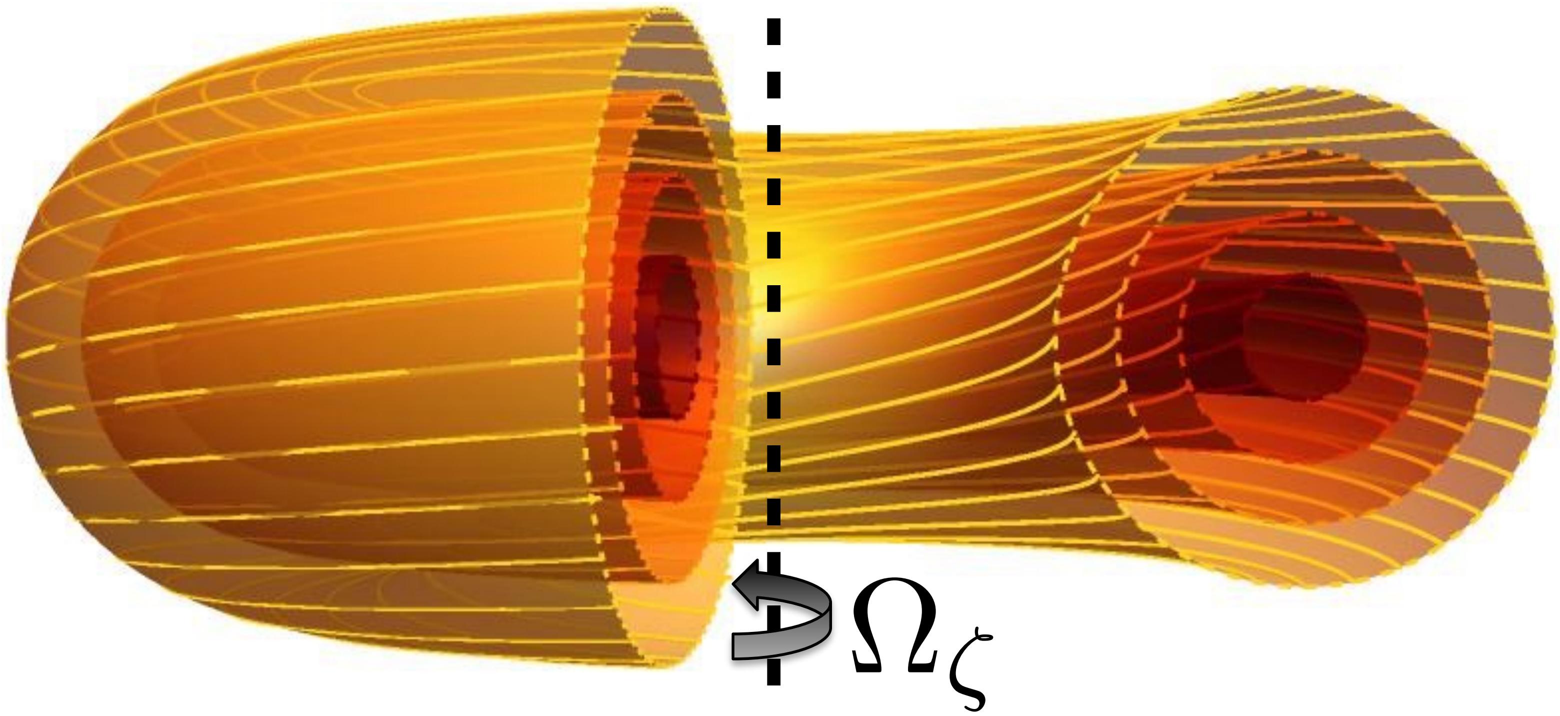
$$\boxed{M_A \equiv \frac{R\Omega_\zeta}{v_{\mathrm{Alfvén}}} \approx 1 - 5\%}^{*} \quad \text{(on-axis)}$$

*multiply by ~10 to get $M_S \equiv \frac{R\Omega_\zeta}{v_{\mathrm{sound}}}$

- DIII-D used $\sim 4\%$ rotation to enable discharges with $\beta_N \approx 1.5\beta_N^{\mathrm{Troyon}}$

Toroidal rotation

Liu et al. *Nucl. Fusion* (2004).



- Each flux surface is free to rotate toroidally because of axisymmetry
- A plasma will start to spin if you push it (neutral beams, RF waves) or if it pushes off itself (“intrinsic rotation” from turbulence)
- Rotation from neutral beams in ITER has a Alfvén Mach number of $\sim 0.3\%$

Intrinsic rotation

- Generated by the turbulent transport of momentum between flux surfaces

- Scales well to large devices and requires no external power

- Solve $\left\langle \Pi \left(\Omega_\zeta, \frac{d\Omega_\zeta}{dr_\psi} \right) \right\rangle_t = 0$ for steady state $\Omega_\zeta \left(r_\psi \right)$:

$$\left\langle \Pi \left(\Omega_\zeta = 0, \frac{d\Omega_\zeta}{dr_\psi} = 0 \right) \right\rangle_t - P_\Pi \Omega_\zeta - D_\Pi \frac{d\Omega_\zeta}{dr_\psi} \approx 0$$

- All we need is for the turbulent transport of momentum in the absence of both rotation and rotation shear **to be non-zero**, i.e

$$\left\langle \Pi \left(0, 0 \right) \right\rangle_t \neq 0$$

"The turbulent transport of momentum in the absence of both rotation and rotation shear **must be zero**…"

Peeters, et al. *Phys. Plasmas* (2005).
Parra, et al. *Phys. Plasmas* (2011).
Sugama, et al. *PPCF* (2011).

“The turbulent transport of momentum in the absence of both rotation and rotation shear must be zero,

for up-down symmetric tokamaks.”

Peeters, et al. *Phys. Plasmas* (2005).
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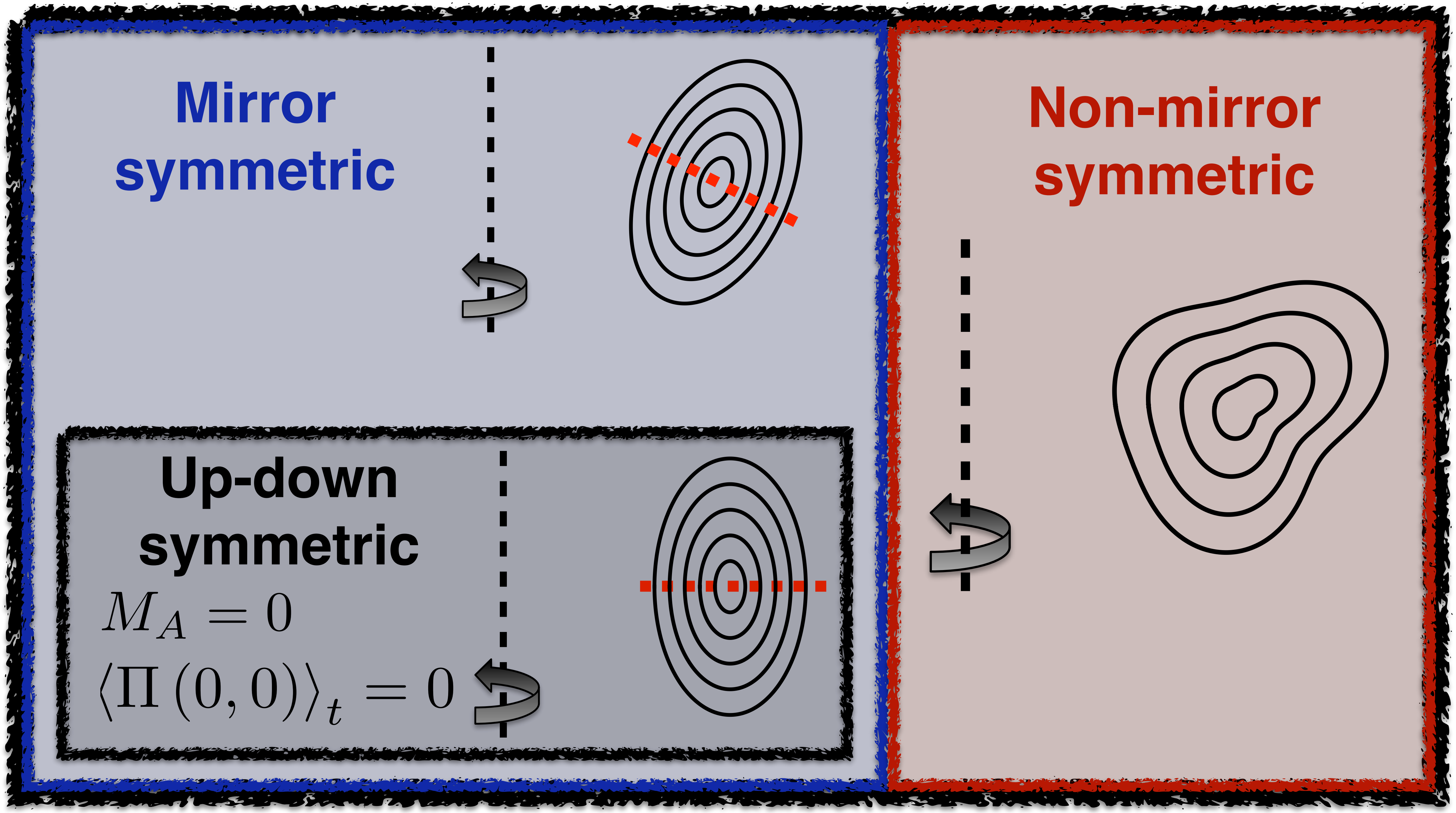
Sugama, et al. *PPCF* (2011).

Outline

Tokamaks



Gyrokinetics

- Governs turbulence in tokamaks:

$$\frac{\partial h_s}{\partial t} + v_{||}\hat{b}\cdot\vec{\nabla}\theta \left.\frac{\partial h_s}{\partial \theta}\right|_{v_{||}} + i\left(k_\psi v_{ds\psi} + k_\alpha v_{ds\alpha}\right) h_s + a_{||s}\frac{\partial h_s}{\partial v_{||}} - \sum_{s'}\langle C_{ss'}^{(l)}\rangle_\varphi + \left\{J_0\left(k_\perp \rho_s\right)\phi, h_s\right\}$$

$$= \frac{Z_s e F_{Ms}}{T_s}\frac{\partial}{\partial t}\left(J_0\left(k_\perp\rho_s\right)\phi\right) - v_{\phi s\psi}F_{Ms}\left[\frac{1}{n_s}\frac{dn_s}{d\psi} + \left(\frac{m_s v^2}{2T_s} - \frac{3}{2}\right)\frac{1}{T_s}\frac{dT_s}{d\psi}\right]$$

where $k_\perp = \sqrt{k_\psi^2\left|\vec{\nabla}\psi\right|^2 + 2k_\psi k_\alpha \vec{\nabla}\psi\cdot\vec{\nabla}\alpha + k_\alpha^2\left|\vec{\nabla}\alpha\right|^2}$

- Allows us to calculate the momentum flux

$$\Pi = 2\pi i I \sum_{k_\psi, k_\alpha} k_\alpha \left\langle \phi\left(k_\psi, k_\alpha\right)\int dv_{||}d\mu\ v_{||}J_0\left(k_\perp\rho_s\right)h_s\left(-k_\psi, -k_\alpha\right)\right\rangle_\psi$$

Gyrokinetics

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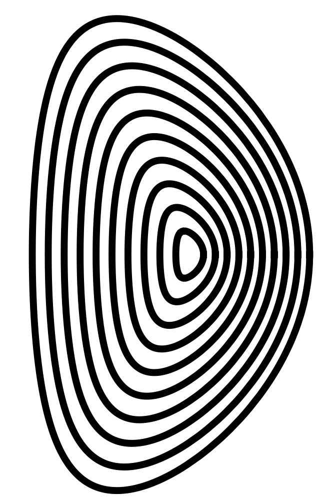

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$$\Pi = 2\pi i I \sum_{k_\psi, k_\alpha} k_\alpha \left\langle \phi\left(k_\psi, k_\alpha\right)\int dv_{||}d\mu\; v_{||}J_0\left(k_\perp \rho_s\right)h_s\left(-k_\psi, -k_\alpha\right)\right\rangle_\psi$$

- Calculate the eight geometric coefficients from MHD equilibrium

Generalization of Miller local equilibrium

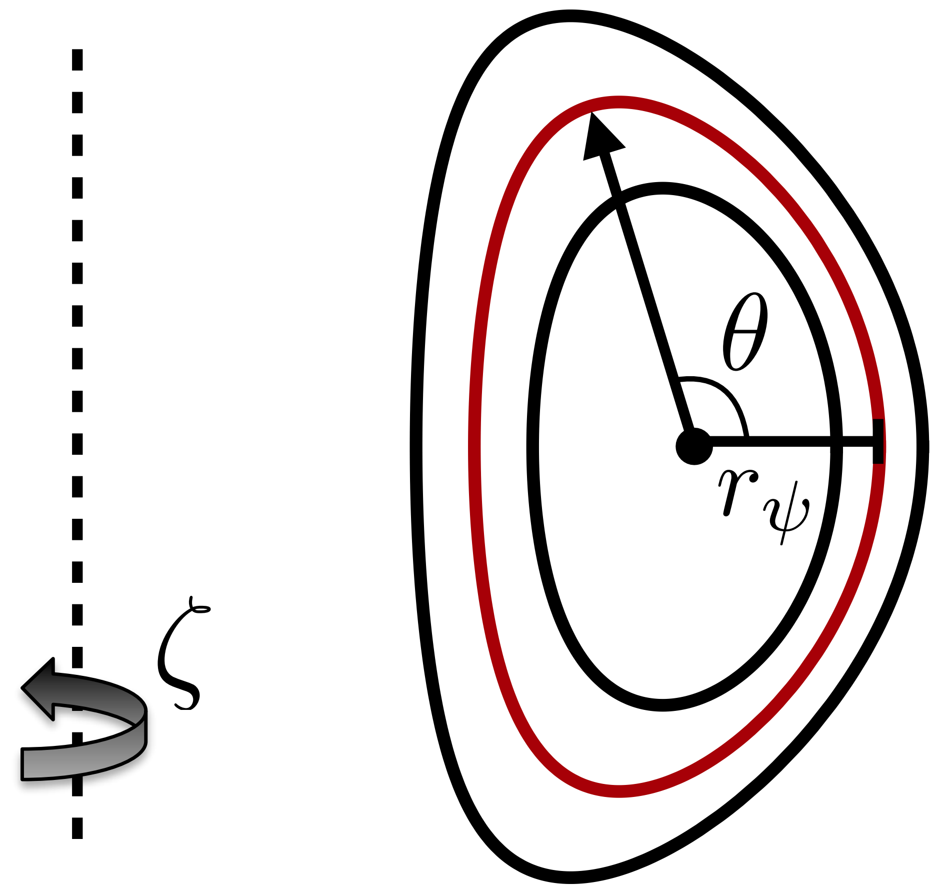
Miller et al. *Phys. Plasmas* (1998).

- Works well with GS2, a local δf gyrokinetic code
- Specify the flux surface of interest as a Fourier decomposition:

$$r_0\left(r_\psi,\theta\right) = r_\psi\left(1 - \sum_m C_m \cos\left(m\left(\theta + \theta_{tm}\right)\right)\right)$$

- Specify how it changes with minor radius:

$$\left.\frac{\partial r_0}{\partial r_\psi}\right|_\theta = 1 - \sum_m C'_m \cos\left(m\left(\theta + \theta'_{tm}\right)\right)$$



Rotation in up-down symmetric tokamaks

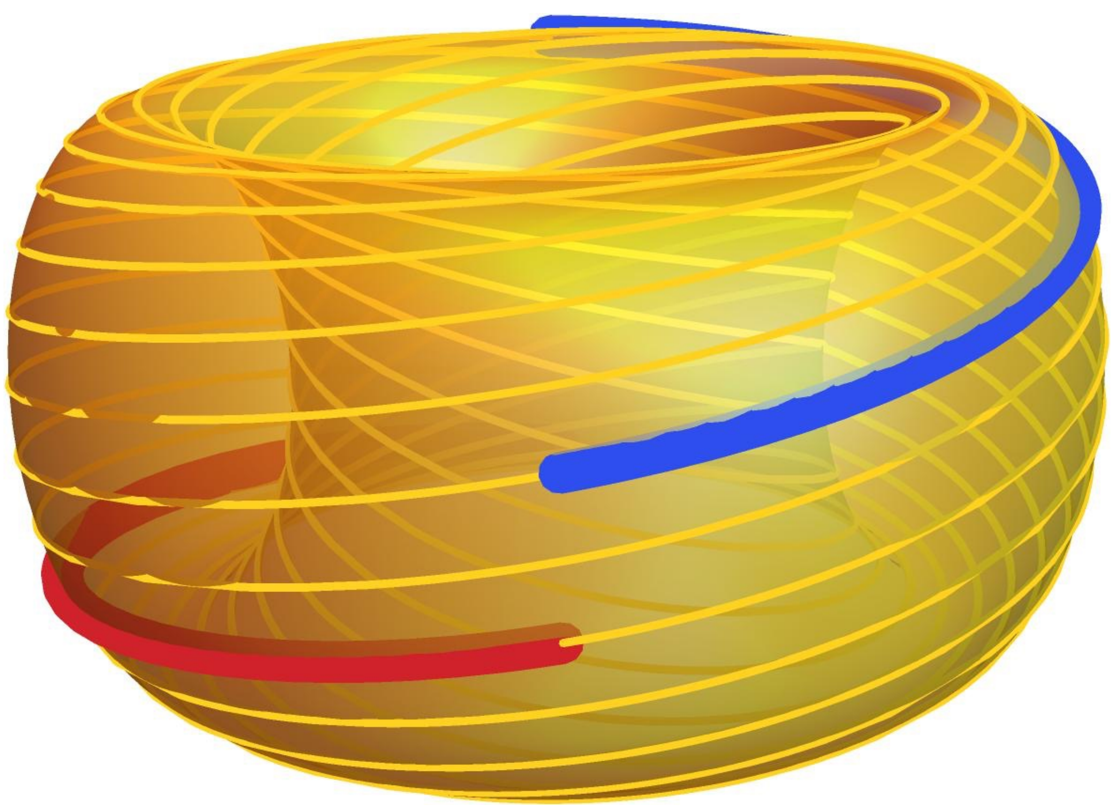
Peeters et al. *PoP* (2005). & Parra et al. *PoP* (2011).

Sugama et al. *PPCF* (2011).

- Negating k_ψ, θ, and $v_{||}$ leads to a second solution of the gyrokinetic eq.

$$\left\{B, \hat{b}\cdot\vec{\nabla}\theta, v_{ds\psi}, v_{ds\alpha}, a_{||s}, \left|\vec{\nabla}\psi\right|^2, \vec{\nabla}\psi\cdot\vec{\nabla}\alpha, \left|\vec{\nabla}\alpha\right|^2\right\}$$
$$\rightarrow \left\{B, \hat{b}\cdot\vec{\nabla}\theta, -v_{ds\psi}, v_{ds\alpha}, -a_{||s}, \left|\vec{\nabla}\psi\right|^2, -\vec{\nabla}\psi\cdot\vec{\nabla}\alpha, \left|\vec{\nabla}\alpha\right|^2\right\}$$

$$h_s\left(k_\psi, k_\alpha, \theta, v_{||}, \mu, t\right) \rightarrow -h_s\left(-k_\psi, k_\alpha, -\theta, -v_{||}, \mu, t\right)$$



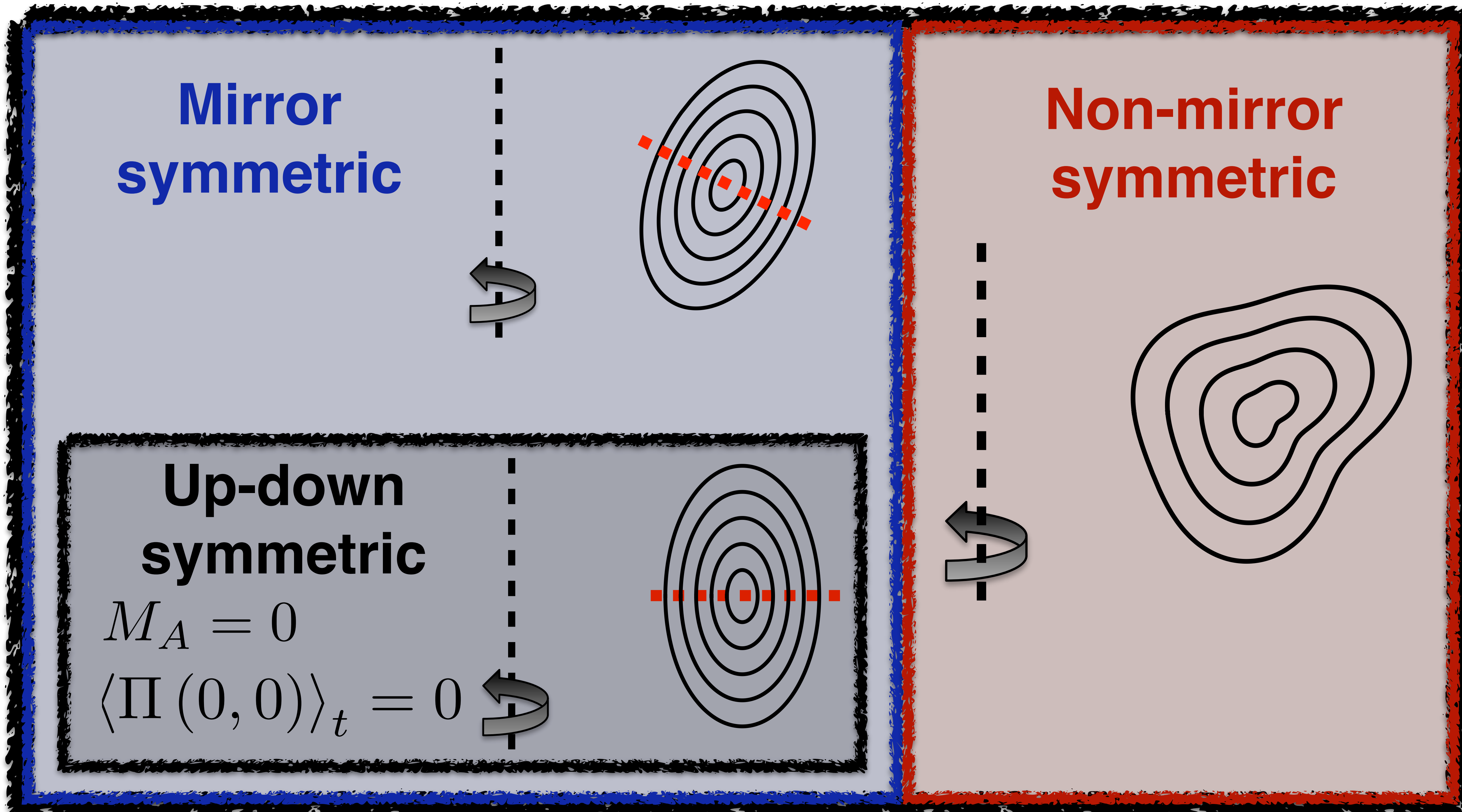
$$\left\langle \Pi\left(0,0\right)\right\rangle_t \rightarrow -\left\langle \Pi\left(0,0\right)\right\rangle_t$$

- Contributions to momentum flux from two symmetric particle trajectories cancel

- Constrains $\left\langle \Pi\left(0,0\right)\right\rangle_t = 0$ to lowest order in $\rho_* \equiv \rho_i/a \ll 1$

Outline

Tokamaks



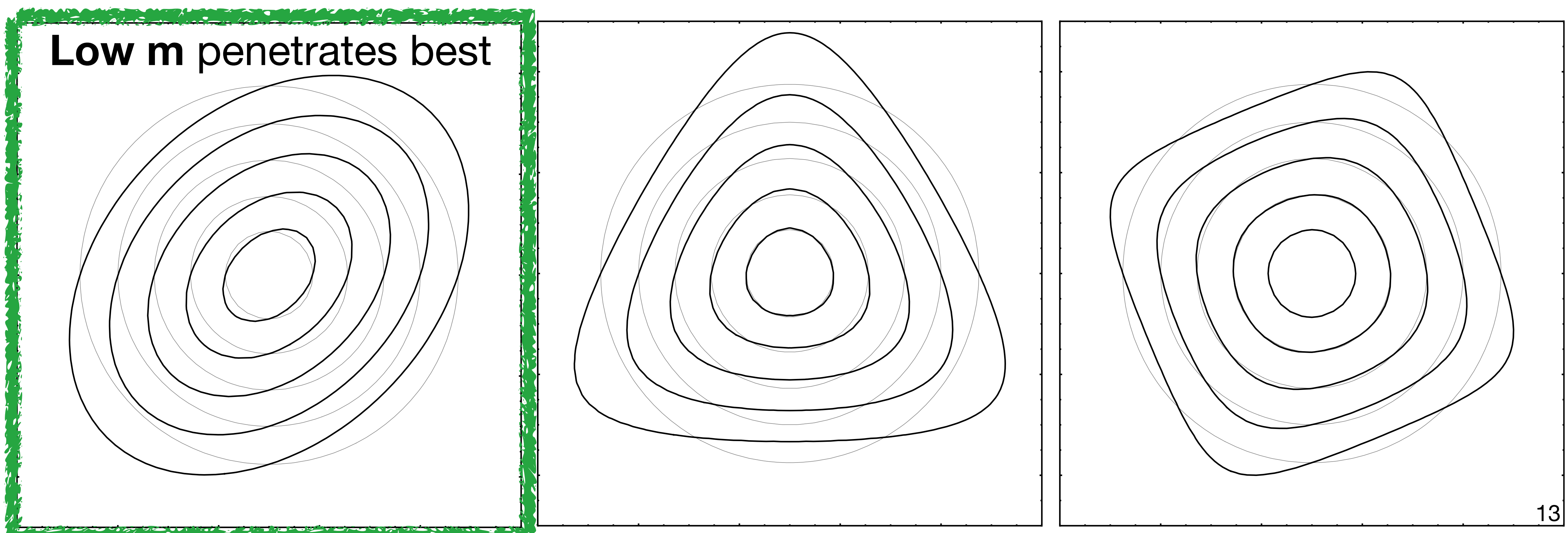
MHD equilibrium of up-down asymmetry

Rodrigues et al. *Nucl. Fusion* (2014).

Ball et al. *PPCF* (2015).

- Grad-Shafranov equation: $R^2 \vec{\nabla} \cdot \left(\frac{\vec{\nabla}\psi}{R^2} \right) = -\mu_0 R^2 \frac{dp}{d\psi} - I \frac{dI}{d\psi}$
- Let the toroidal current be constant, i.e. $-\mu_0 R^2 \frac{dp}{d\psi} - I \frac{dI}{d\psi} = c_0$
- To lowest order in aspect ratio, solutions are cylindrical harmonics:

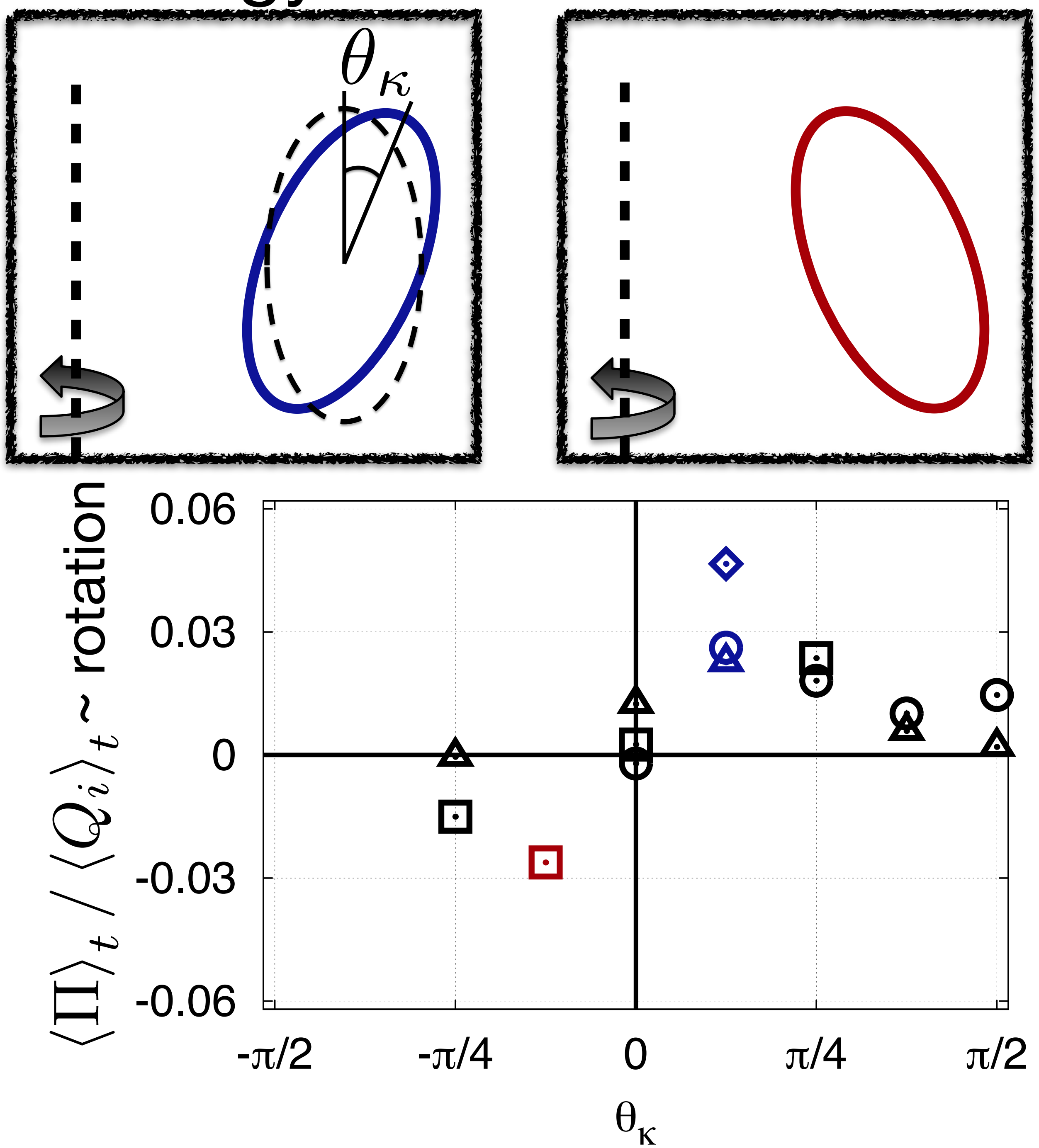
m=2 mode m=3 mode m=4 mode



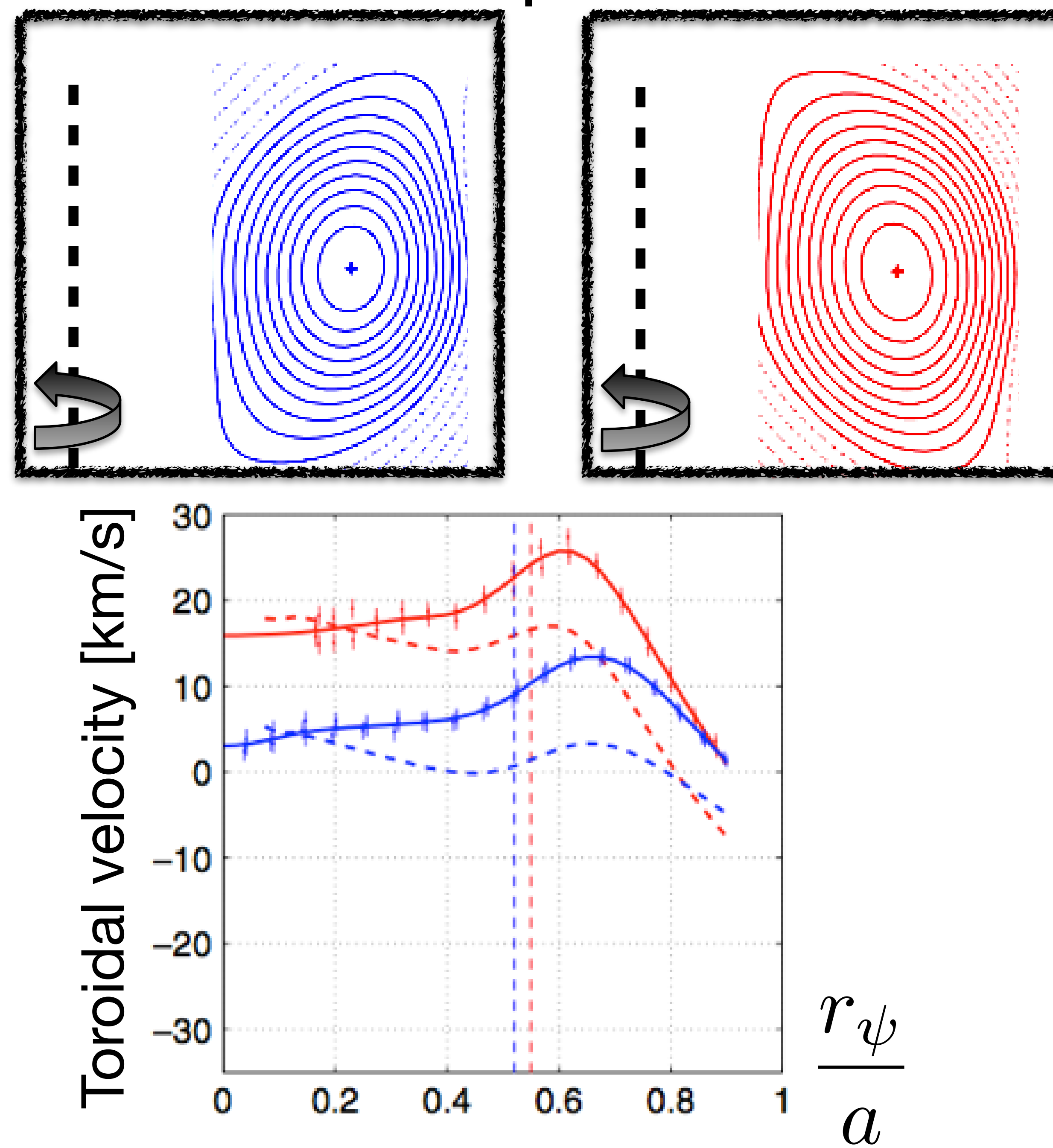
Tilted elliptical geometry (m=2)

Ball et al. *PPCF* (2014).
Camenen et al. *PPCF* (2010).

GS2 gyrokinetic simulations



TCV experiment



Estimating the Alfvén Mach number

Ball et al. *PPCF* (2014).
Peeters et al. *PRL* (2007).

ignore

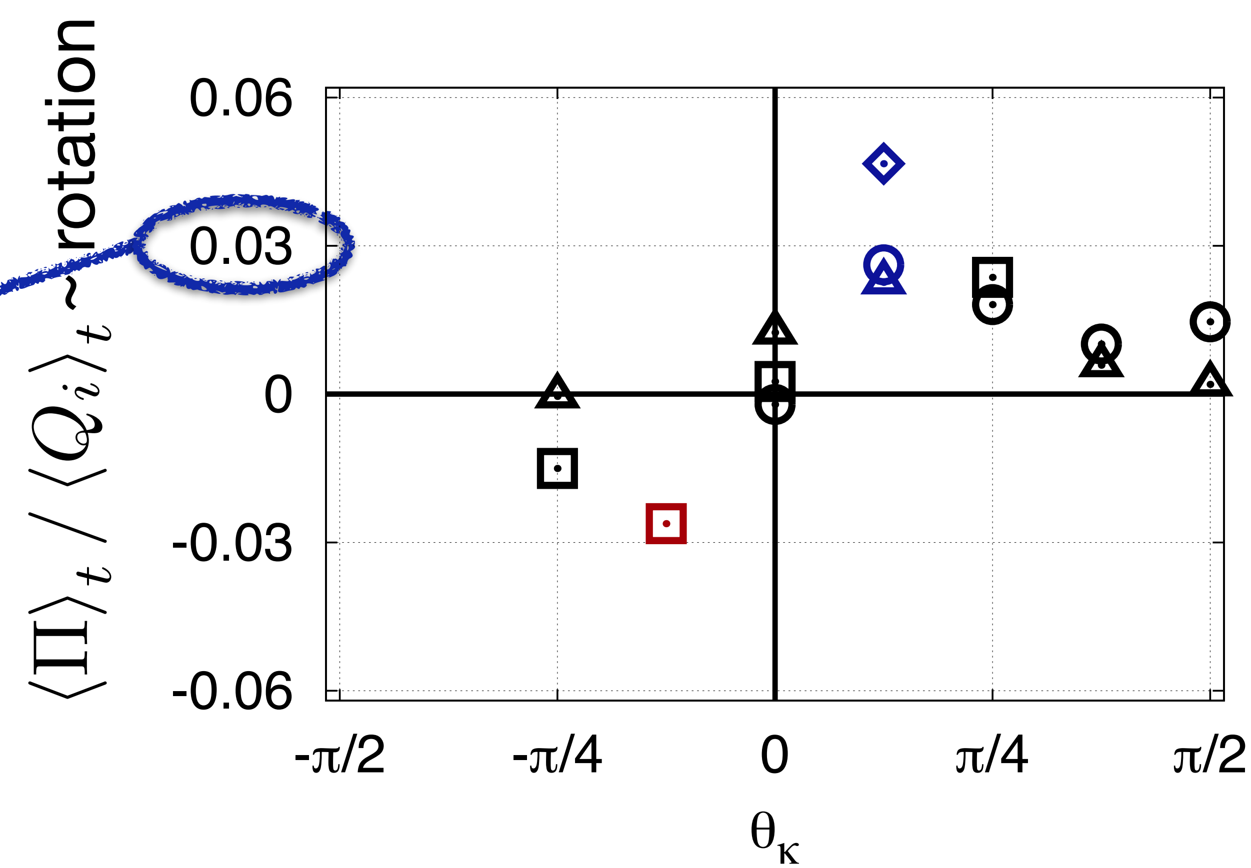
$$\langle \Pi(0,0) \rangle_t - P_{\Pi} \Omega_\zeta - D_{\Pi} \frac{d\Omega_\zeta}{dr_\psi} \approx 0 \qquad \langle Q_i \rangle_t \approx -D_Q \frac{dT_i}{dr_\psi}$$

$$Pr \equiv \frac{D_\Pi}{D_Q} \approx 1 \approx \text{constant}$$

$$M_A \approx \frac{\sqrt{2\beta_T}}{Pr} \frac{\langle \Pi(0,0) \rangle_t}{\langle Q_i \rangle_t}$$

$$M_A \approx 1.5\%$$

(in ITER)

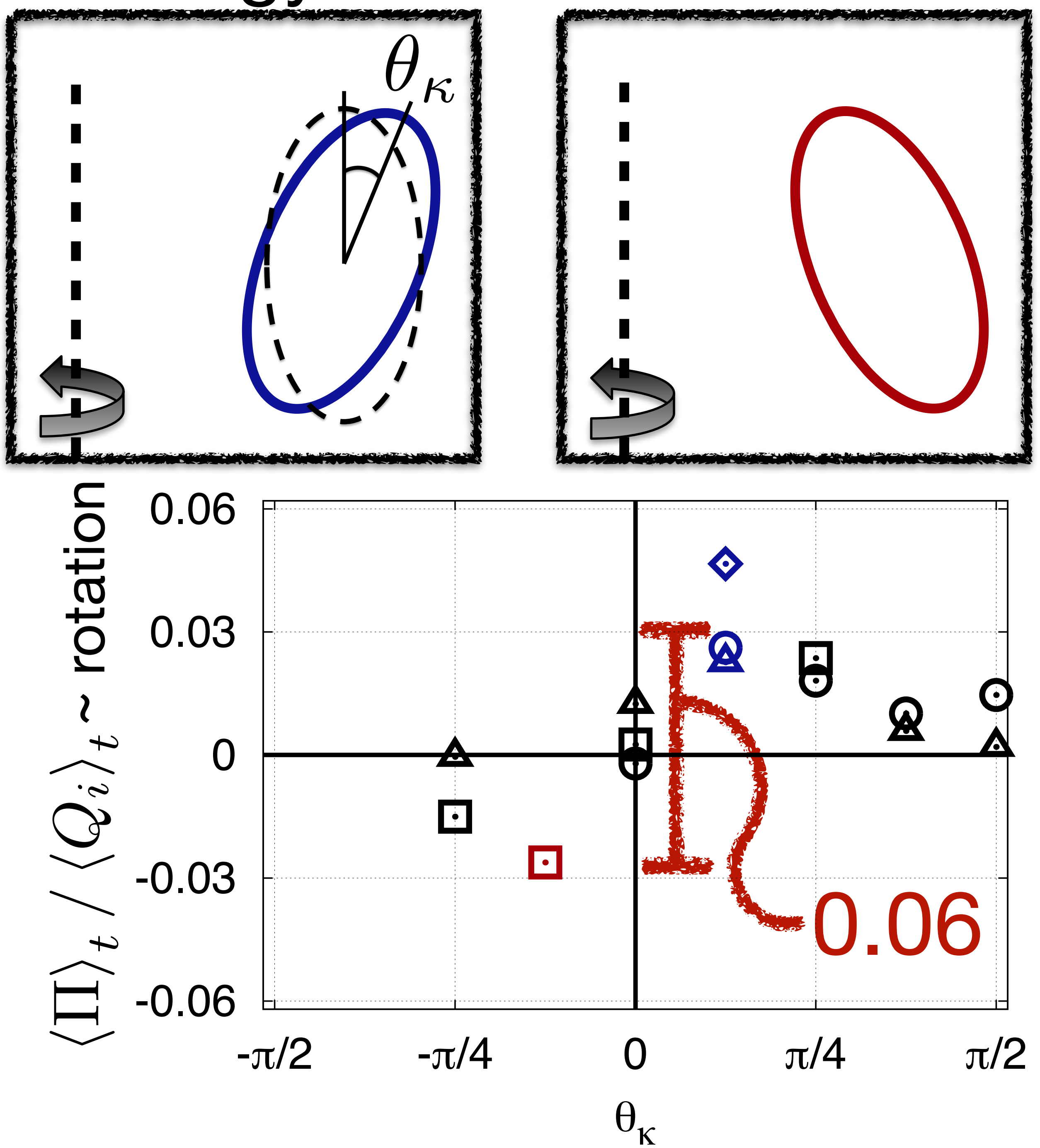


- Ignoring pinch is conservative, may enhance rotation by a factor of 3

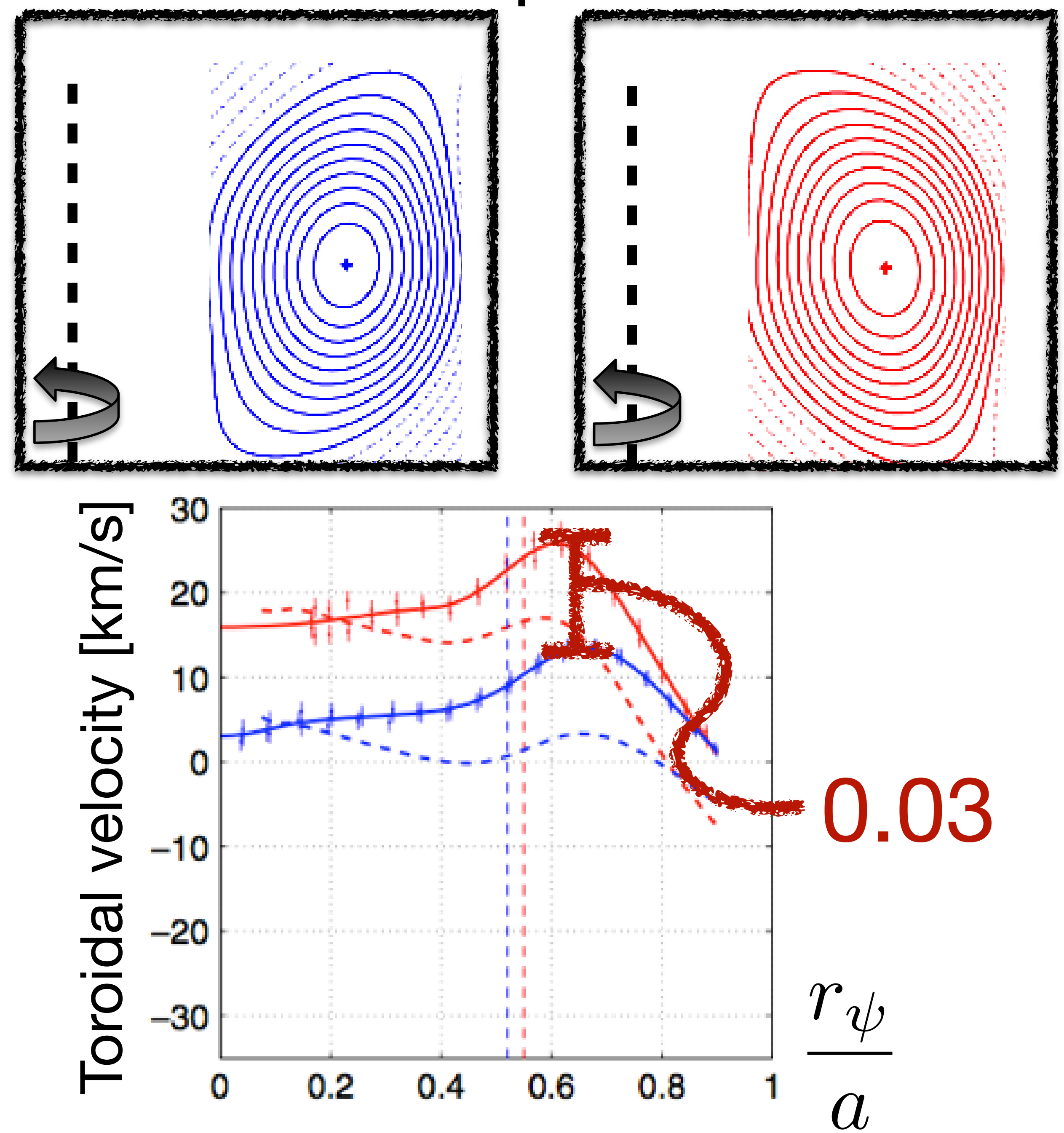
Tilted elliptical geometry (m=2)

Ball et al. *PPCF* (2014).
Camenen et al. *PPCF* (2010).

GS2 gyrokinetic simulations

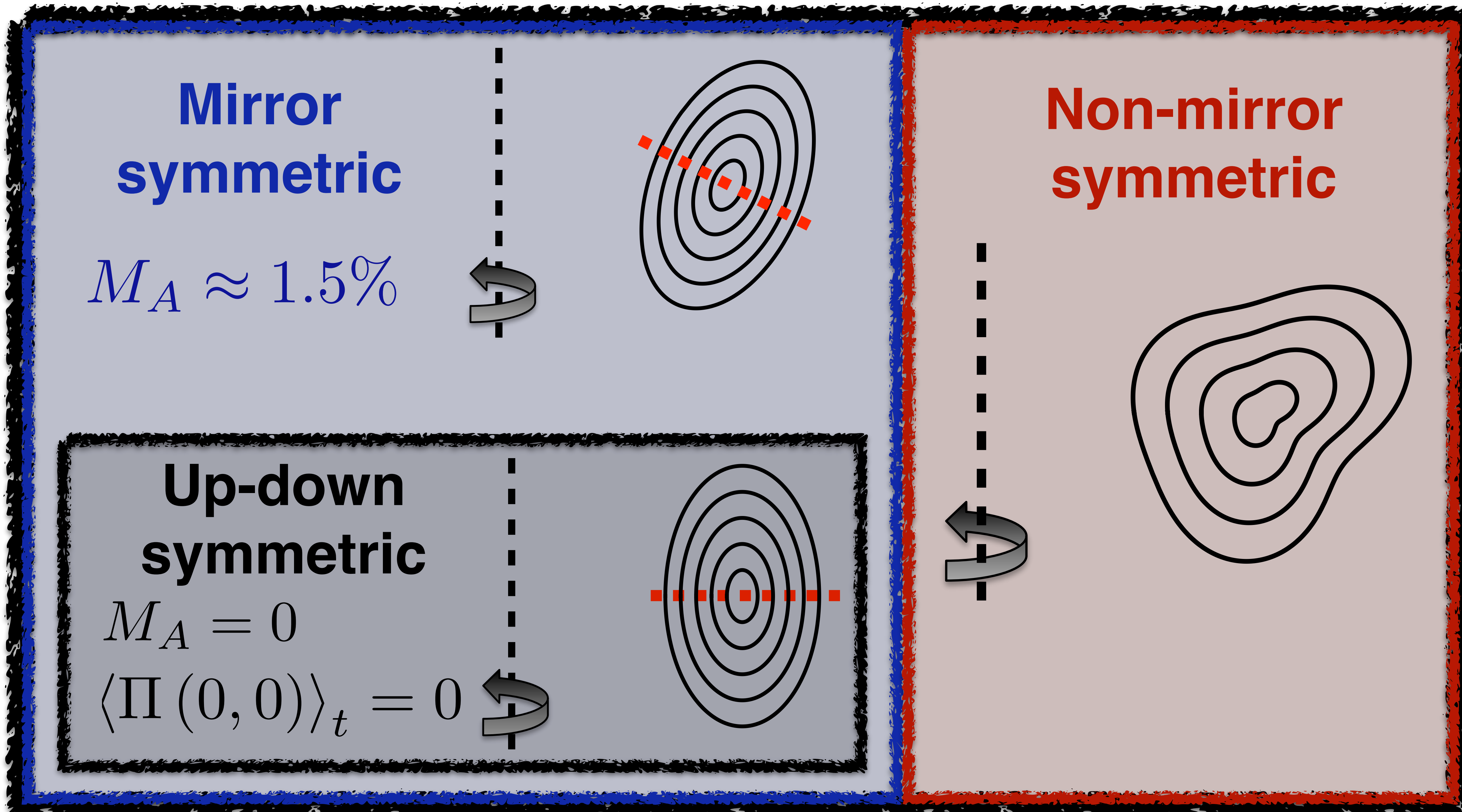


TCV experiment



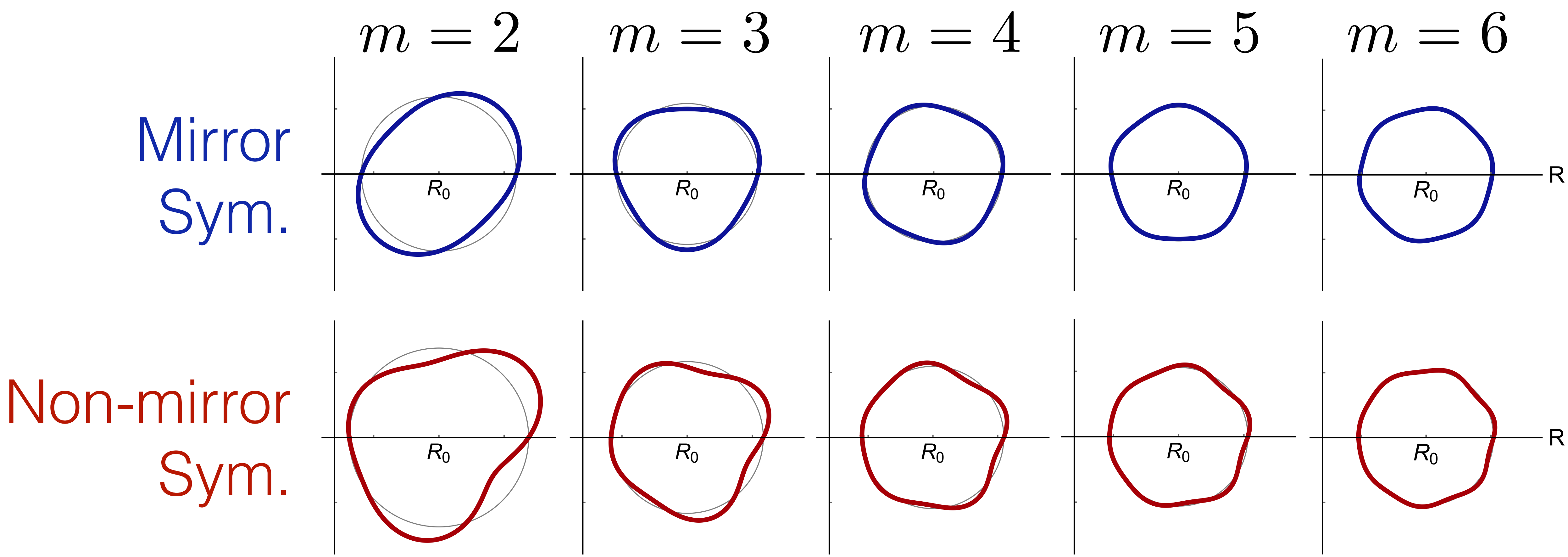
Outline

Tokamaks



Scanning and expanding in $m \gg 1$

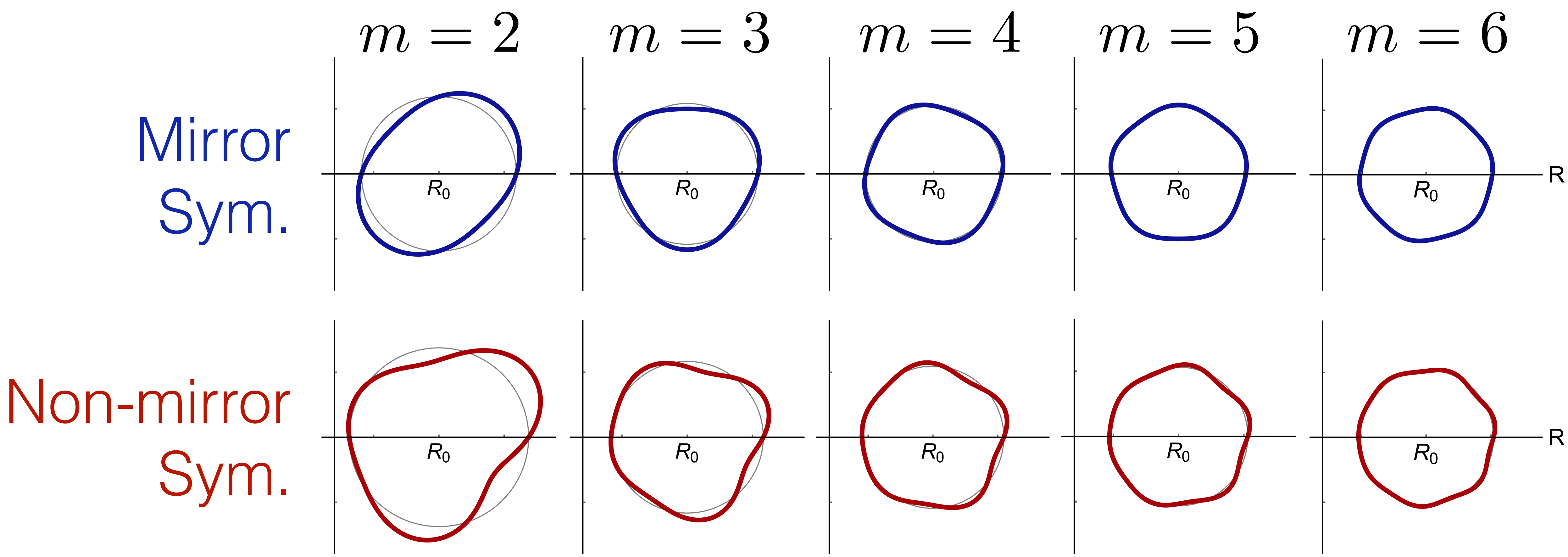
Ball, et al. *PPCF* (in prep).



- Counterintuitive because we just said low m penetrates better
- High m flux surfaces could transport momentum better
- Theoretical tool to allow analytic results

Scaling of momentum flux with $m \gg 1$

Ball, et al. *PPCF* (in prep).

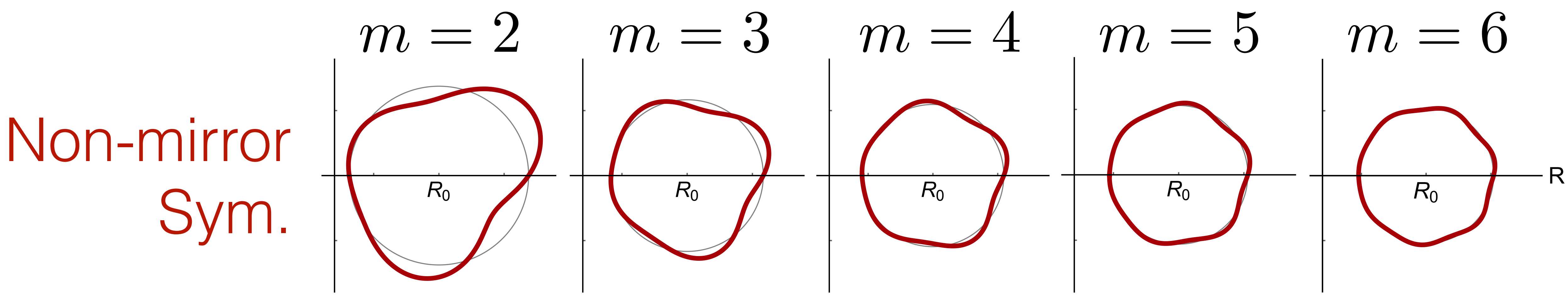


- Expect that $\langle \Pi(0,0) \rangle_t \sim m^{-\alpha}$, since the turbulent mode averages over the fast variation

- $\langle \Pi(0,0) \rangle_t \sim m^{-\alpha}$ is true for non-mirror symmetric geometries, but not for mirror symmetric geometries

Non-mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).



- Created using two modes, $m \sim n \gg 1$, with distinct tilts $\theta_{tm}, \ \theta_{tn}$
- Assume reasonable shaping: $C_m \sim C_n \sim m^{-2} \ll 1$
- Calculate geometric coefficients order-by-order in $m \gg 1$
- Look for beating between fast shaping effects (creates symmetry-breaking on the connection length)

Non-mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).

- Drift coefficient, $v_{ds\alpha}$, dominates (symmetry-breaking enters to lowest order in aspect ratio and is not a finite gyroradius effect)

Non-mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).

- Drift coefficient, $v_{ds\alpha}$, dominates (symmetry-breaking enters to lowest order in aspect ratio and is not a finite gyroradius effect)

$$
\begin{aligned}
v_{ds\alpha} = & \frac{B_0}{R_0 \Omega_s} \frac{dr_\psi}{d\psi} \left(\cos\left(\theta\right) + \hat{s}\theta \sin\left(\theta\right)\right) \\
& + \frac{B_0}{2 R_0 \Omega_s} \frac{dr_\psi}{d\psi} \left[\left(m^3 C_m^2 + n^3 C_n^2\right) \theta \sin\left(\theta\right)\right. \\
& - \hat{s}\theta \cos\left(\theta\right) \left(m C_m \sin\left(m\left(\theta - \theta_{tm}\right)\right) + n C_n \sin\left(n\left(\theta - \theta_{tn}\right)\right)\right) \\
& + \hat{s}\theta \sin\left(\theta\right) \left(m C_m \cos\left(m\left(\theta - \theta_{tm}\right)\right) + n C_n \cos\left(n\left(\theta - \theta_{tn}\right)\right)\right) \\
& + mn \frac{m+n}{n-m} C_m C_n \sin\left(\theta\right) \\
& \left.\times \left(\sin\left(\left(n-m\right)\theta\right) \cos\left(m\left(\theta_{tm} - \theta_{tn}\right)\right) - \cos\left(\left(n-m\right)\theta\right) \sin\left(m\left(\theta_{tm} - \theta_{tn}\right)\right)\right)\right]
\end{aligned}
$$

Non-mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).

- Drift coefficient, $v_{ds\alpha}$, dominates (symmetry-breaking enters to lowest order in aspect ratio and is not a finite gyroradius effect)

$$v_{ds\alpha} = \frac{B_0}{R_0 \Omega_s} \frac{dr_\psi}{d\psi} \left(\cos(\theta) + \hat{s}\theta \sin(\theta) \right)$$

up-down sym.

$$+ \frac{B_0}{2R_0\Omega_s} \frac{dr_\psi}{d\psi} \left[\left(m^3 C_m^2 + n^3 C_n^2 \right) \theta \sin(\theta) \right.$$

$$- \hat{s}\theta \cos(\theta) \left(m C_m \sin(m(\theta - \theta_{tm})) + n C_n \sin(n(\theta - \theta_{tn})) \right)$$

$$+ \hat{s}\theta \sin(\theta) \left(m C_m \cos(m(\theta - \theta_{tm})) + n C_n \cos(n(\theta - \theta_{tn})) \right)$$

0

$$+ mn \frac{m+n}{n-m} C_m C_n \sin(\theta)$$

$$\left. \times \left(\sin((n-m)\theta) \cos(m(\theta_{tm} - \theta_{tn})) - \cos((n-m)\theta) \sin(m(\theta_{tm} - \theta_{tn})) \right) \right]$$

breaks symmetry

- Turbulence averages over high m variation

Non-mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).

- Drift coefficient, $v_{ds\alpha}$, dominates (symmetry-breaking enters to lowest order in aspect ratio and is not a finite gyroradius effect)

$$
\begin{aligned}
v_{ds\alpha} = & \frac{B_0}{R_0\Omega_s}\frac{dr_\psi}{d\psi}\left(\cos(\theta) + \hat{s}\theta\sin(\theta)\right) \\
& + \frac{B_0}{2R_0\Omega_s}\frac{dr_\psi}{d\psi}\left[\left(m^3C_m^2 + n^3C_n^2\right)\theta\sin(\theta)\right. \\
& - \hat{s}\theta\cos(\theta)\left(mC_m\sin\left(m\left(\theta-\theta_{tm}\right)\right) + nC_n\sin\left(n\left(\theta-\theta_{tn}\right)\right)\right) \\
& + \hat{s}\theta\sin(\theta)\left(mC_m\cos\left(m\left(\theta-\theta_{tm}\right)\right) + nC_n\cos\left(n\left(\theta-\theta_{tn}\right)\right)\right) \\
& + mn\frac{m+n}{n-m}C_mC_n\sin(\theta) \\
& \left.\times\left(\sin\left((n-m)\theta\right)\cos\left(m\left(\theta_{tm}-\theta_{tn}\right)\right) - \cos\left((n-m)\theta\right)\sin\left(m\left(\theta_{tm}-\theta_{tn}\right)\right)\right)\right]
\end{aligned}
$$

up-down sym.

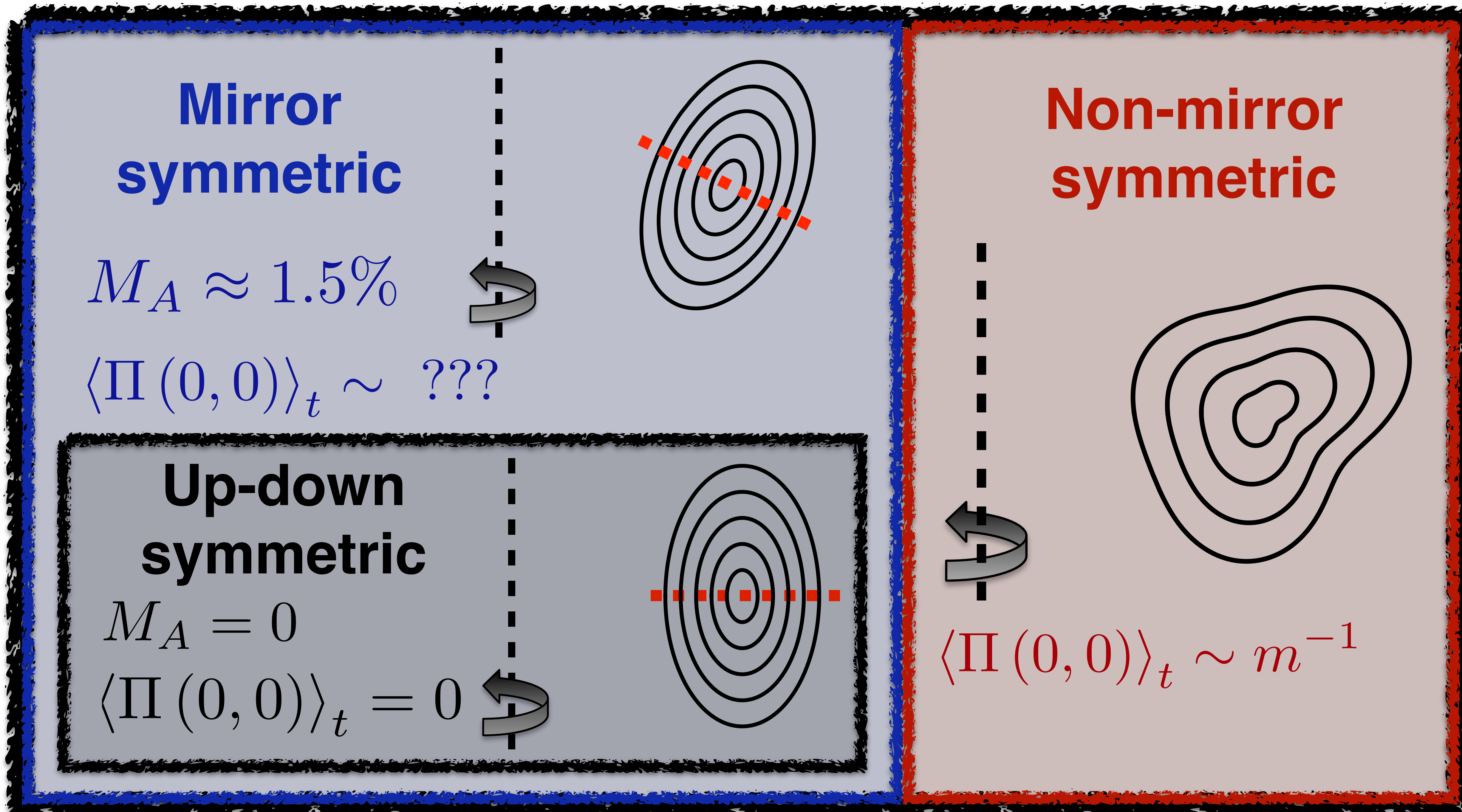
$\rightarrow 0$

$\sim m^{-1}$

- Turbulence averages over high m variation

Outline

Tokamaks



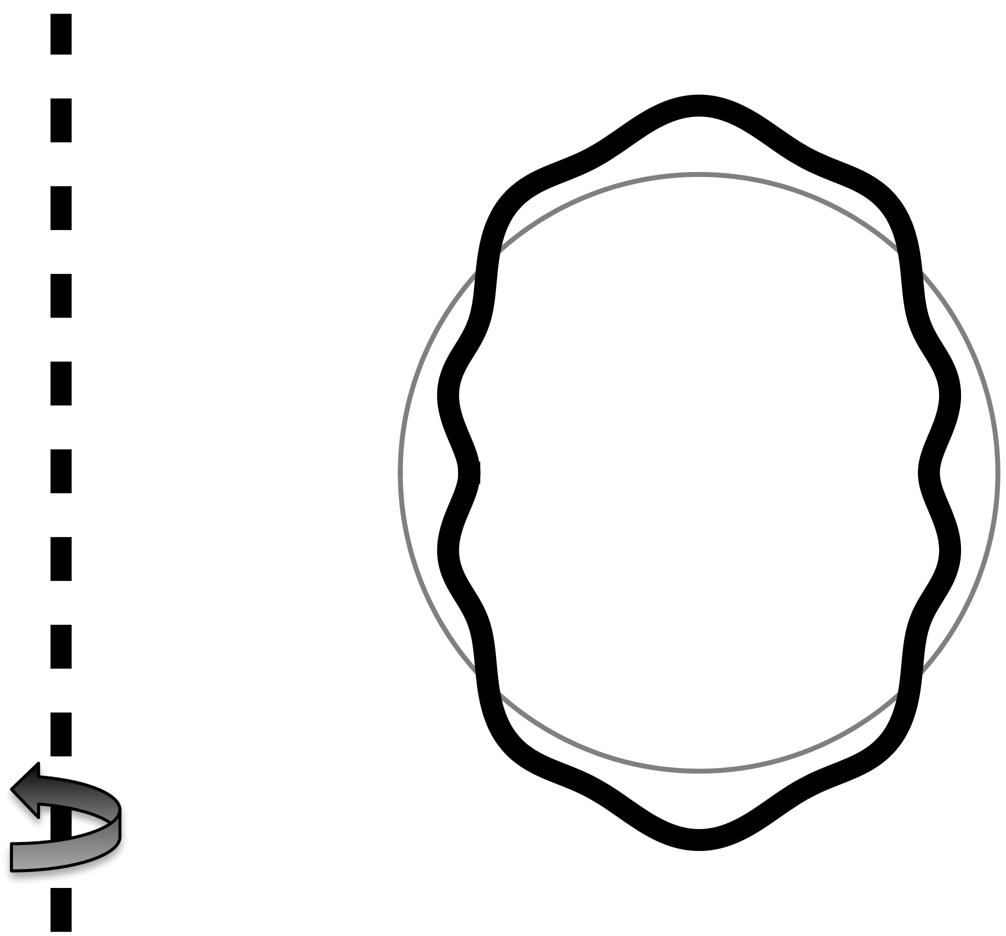
Mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).

- Specify geometry, separating a fast poloidal coordinate $z \equiv m\theta$ from θ

Up-down symmetric

e.g. $r_0^{\mathrm{ud}}(\theta, z) = r_{\psi 0}\left(1 - C_2 \cos(2\theta) - C_m \cos(\underbrace{(m\theta)}_{z})\right)$



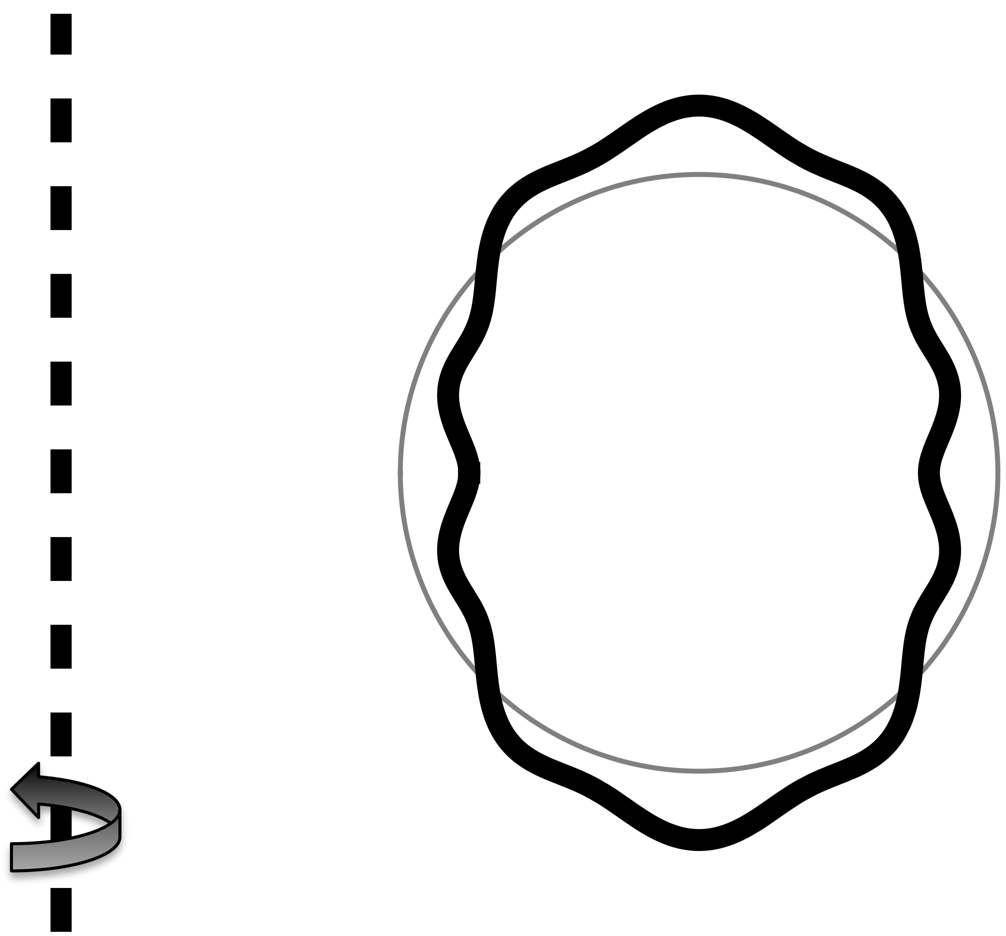
Mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).

- Create a second geometry by tilting the fast flux surface shaping

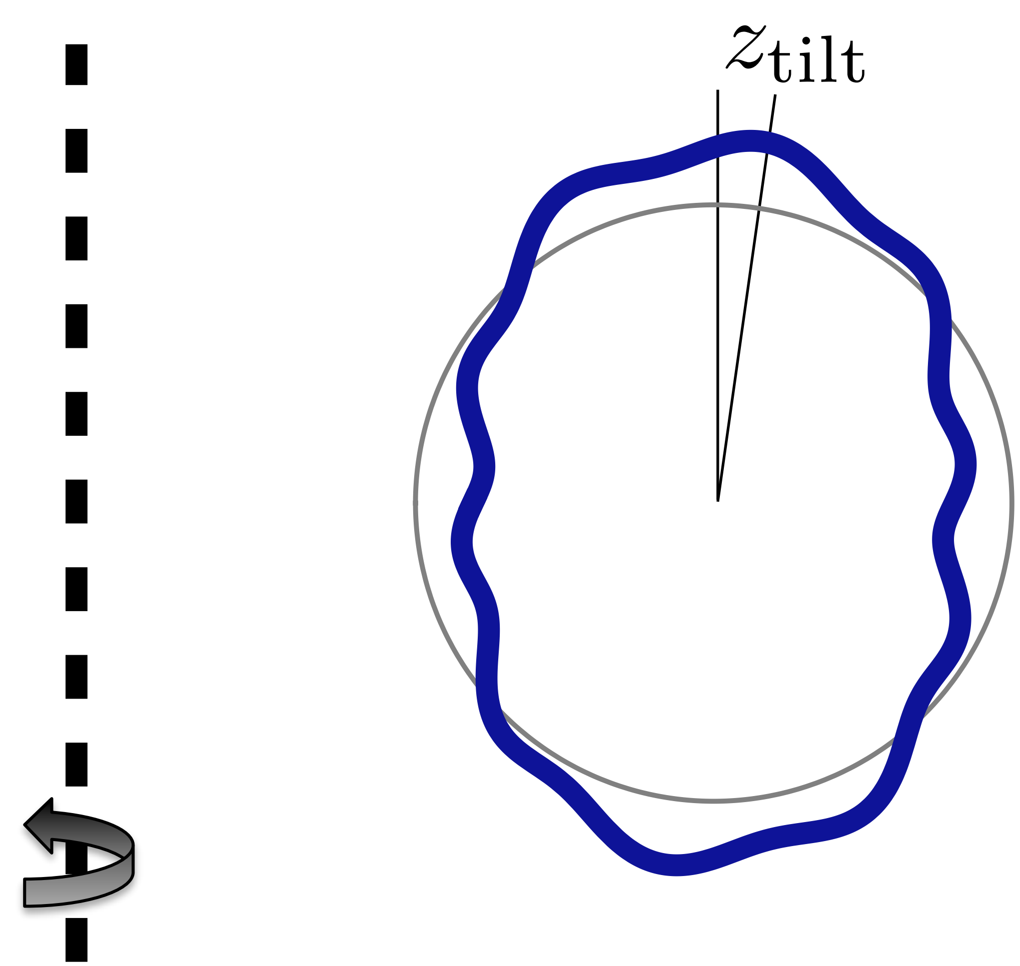
Up-down symmetric

$$r_0^{\mathrm{ud}}(\theta, z)$$



Mirror symmetric

$$r_0^{\mathrm{mir}}(\theta, z) = r_0^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$$



Mirror symmetric geometries with $m \gg 1$

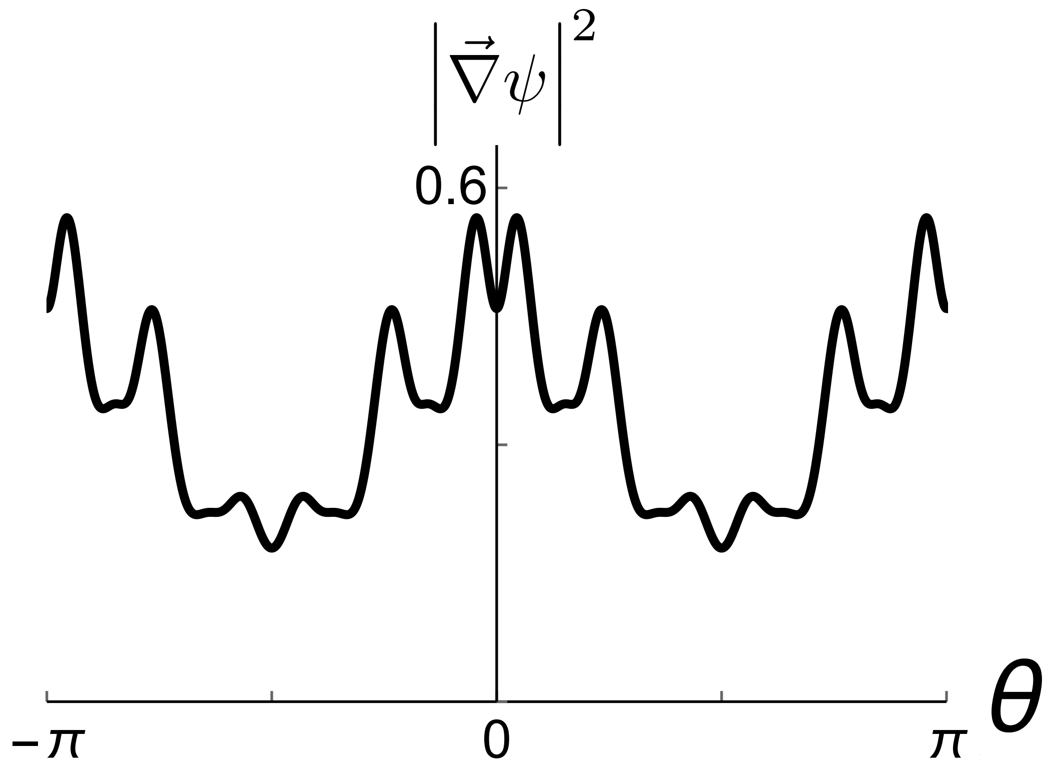
Ball, et al. *PPCF* (in prep).

- Look for differences in the geometric coefficients (critical and non-trivial)

Up-down symmetric

$$r_0^{\mathrm{ud}}(\theta, z)$$

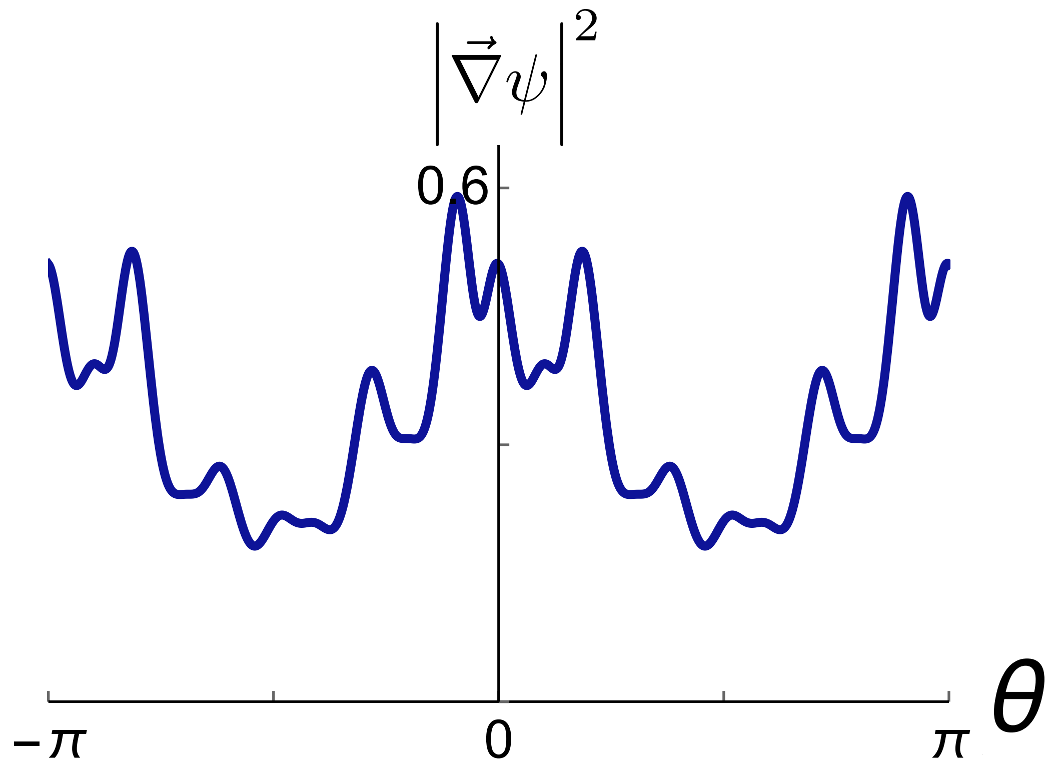
$$Q_{\mathrm{geo}}^{\mathrm{ud}}(\theta, z)$$



Mirror symmetric

$$r_0^{\mathrm{mir}}(\theta, z) = r_0^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$$

$$Q_{\mathrm{geo}}^{\mathrm{mir}}(\theta, z) = Q_{\mathrm{geo}}^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$$



Mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).

- The geometric coefficients are the only way the geometry enters, so…

Up-down symmetric	**Mirror symmetric**
$r_0^{\text{ud}}(\theta, z)$	$r_0^{\text{mir}}(\theta, z) = r_0^{\text{ud}}(\theta, z + z_{\text{tilt}})$
$Q_{\text{geo}}^{\text{ud}}(\theta, z)$	$Q_{\text{geo}}^{\text{mir}}(\theta, z) = Q_{\text{geo}}^{\text{ud}}(\theta, z + z_{\text{tilt}})$
$h_s^{\text{ud}}(\theta, z)$	$h_s^{\text{mir}}(\theta, z) = h_s^{\text{ud}}(\theta, z + z_{\text{tilt}})$
$\phi^{\text{ud}}(\theta, z)$	$\phi^{\text{mir}}(\theta, z) = \phi^{\text{ud}}(\theta, z + z_{\text{tilt}})$

Mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).

- Define π_ζ such that $\Pi = \left\langle \left\langle \pi_\zeta \left(\theta, z\right) \right\rangle_z \right\rangle_\psi$

Up-down symmetric	**Mirror symmetric**
$r_0^{\text{ud}}\left(\theta, z\right)$	$r_0^{\text{mir}}\left(\theta, z\right) = r_0^{\text{ud}}\left(\theta, z + z_{\text{tilt}}\right)$
$Q_{\text{geo}}^{\text{ud}}\left(\theta, z\right)$	$Q_{\text{geo}}^{\text{mir}}\left(\theta, z\right) = Q_{\text{geo}}^{\text{ud}}\left(\theta, z + z_{\text{tilt}}\right)$
$h_s^{\text{ud}}\left(\theta, z\right)$	$h_s^{\text{mir}}\left(\theta, z\right) = h_s^{\text{ud}}\left(\theta, z + z_{\text{tilt}}\right)$
$\phi^{\text{ud}}\left(\theta, z\right)$	$\phi^{\text{mir}}\left(\theta, z\right) = \phi^{\text{ud}}\left(\theta, z + z_{\text{tilt}}\right)$
$\pi_\zeta^{\text{ud}}\left(\theta, z\right)$	$\pi_\zeta^{\text{mir}}\left(\theta, z\right) = \pi_\zeta^{\text{ud}}\left(\theta, z + z_{\text{tilt}}\right)$

Mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).

- The z-average in $\Pi = \left\langle \left\langle \pi_\zeta\left(\theta, z\right)\right\rangle_z \right\rangle_\psi$ doesn't care about the tilt

Up-down symmetric	**Mirror symmetric**
$r_0^{\mathrm{ud}}\left(\theta, z\right)$	$r_0^{\mathrm{mir}}\left(\theta, z\right) = r_0^{\mathrm{ud}}\left(\theta, z + z_{\mathrm{tilt}}\right)$
$Q_{\mathrm{geo}}^{\mathrm{ud}}\left(\theta, z\right)$	$Q_{\mathrm{geo}}^{\mathrm{mir}}\left(\theta, z\right) = Q_{\mathrm{geo}}^{\mathrm{ud}}\left(\theta, z + z_{\mathrm{tilt}}\right)$
$h_s^{\mathrm{ud}}\left(\theta, z\right)$	$h_s^{\mathrm{mir}}\left(\theta, z\right) = h_s^{\mathrm{ud}}\left(\theta, z + z_{\mathrm{tilt}}\right)$
$\phi^{\mathrm{ud}}\left(\theta, z\right)$	$\phi^{\mathrm{mir}}\left(\theta, z\right) = \phi^{\mathrm{ud}}\left(\theta, z + z_{\mathrm{tilt}}\right)$
$\pi_\zeta^{\mathrm{ud}}\left(\theta, z\right)$	$\pi_\zeta^{\mathrm{mir}}\left(\theta, z\right) = \pi_\zeta^{\mathrm{ud}}\left(\theta, z + z_{\mathrm{tilt}}\right)$
$\Pi^{\mathrm{ud}} = 0$	$\Pi^{\mathrm{mir}} = 0?$

Mirror symmetric geometries with $m \gg 1$

Ball, et al. *PPCF* (in prep).

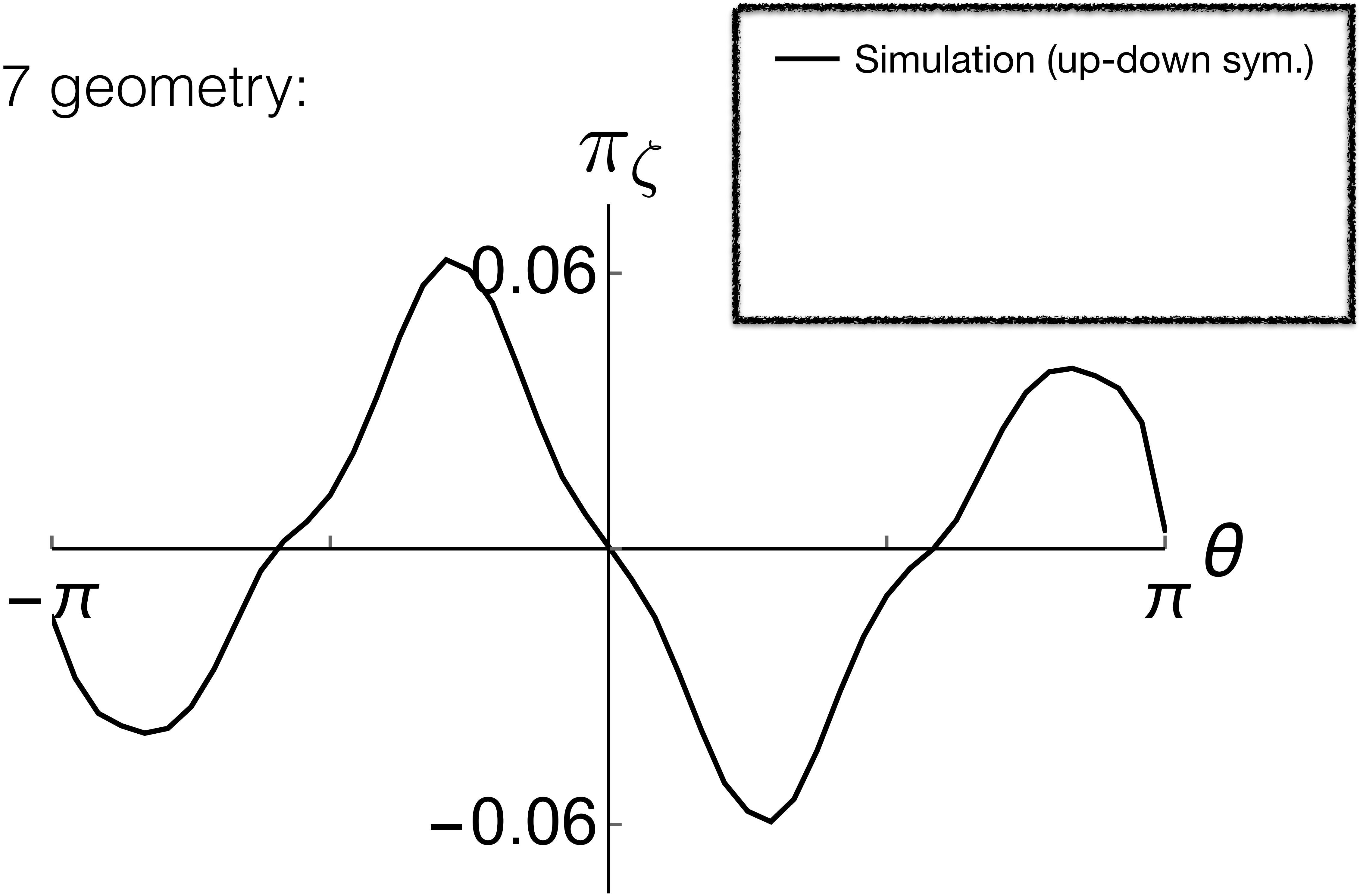
- We relied on expansion in $m \gg 1$ to separate poloidal scales

Up-down symmetric	**Mirror symmetric**
$r_0^{\mathrm{ud}}(\theta, z)$	$r_0^{\mathrm{mir}}(\theta, z) = r_0^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$
$Q_{\mathrm{geo}}^{\mathrm{ud}}(\theta, z)$	$Q_{\mathrm{geo}}^{\mathrm{mir}}(\theta, z) = Q_{\mathrm{geo}}^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$
$h_s^{\mathrm{ud}}(\theta, z)$	$h_s^{\mathrm{mir}}(\theta, z) = h_s^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$
$\phi^{\mathrm{ud}}(\theta, z)$	$\phi^{\mathrm{mir}}(\theta, z) = \phi^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$
$\pi_\zeta^{\mathrm{ud}}(\theta, z)$	$\pi_\zeta^{\mathrm{mir}}(\theta, z) = \pi_\zeta^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$
$\Pi^{\mathrm{ud}} = 0$	~~$\Pi^{\mathrm{mir}} = 0?$~~
	$\Pi \lesssim \exp(-\alpha m)$

Mom. flux distribution, testing $\pi_\zeta^{\mathrm{mir}}(\theta, z) = \pi_\zeta^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$

Ball, et al. *PPCF* (in prep).

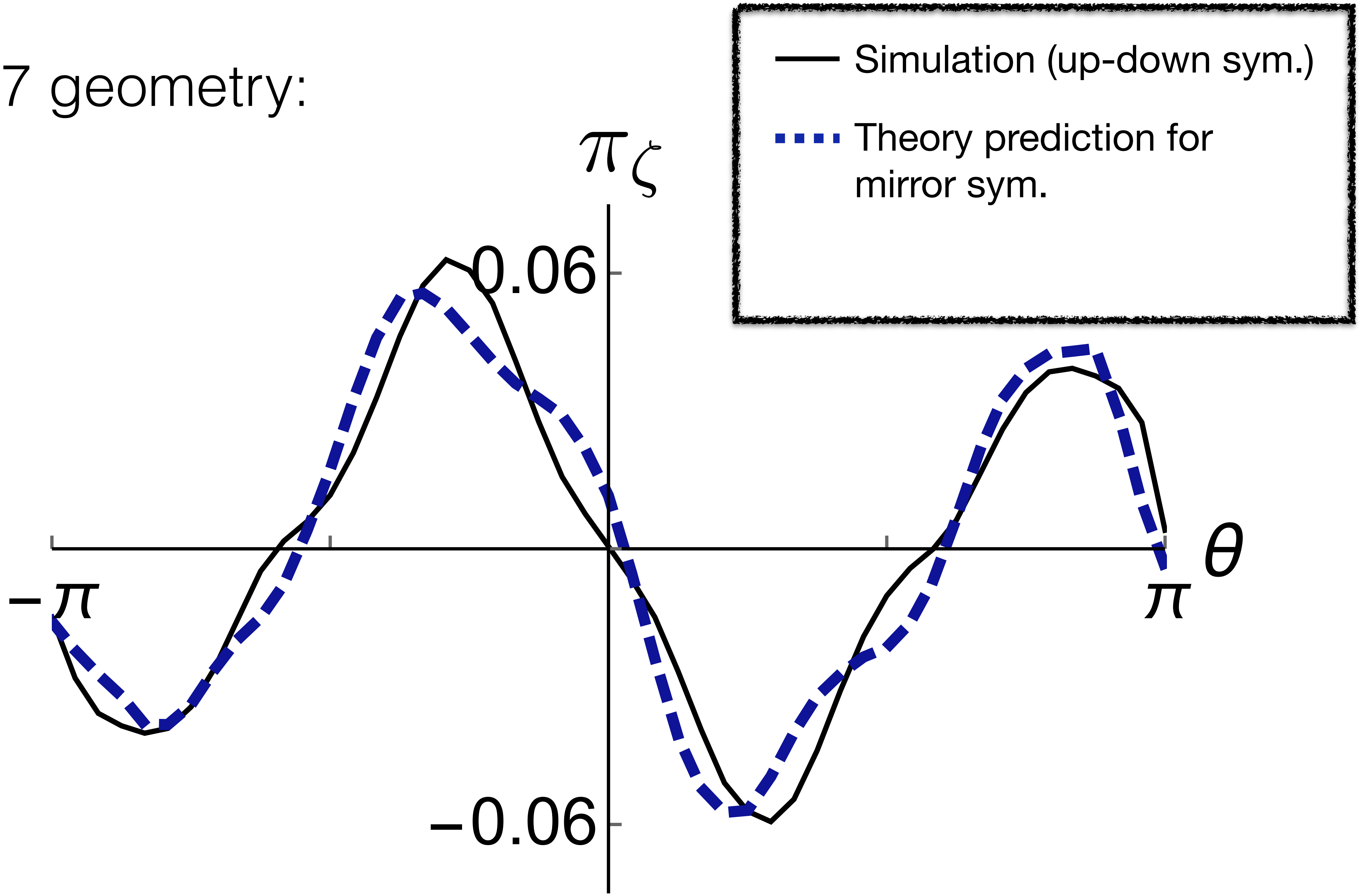
m=7 geometry:



Mom. flux distribution, testing $\pi_\zeta^{\rm mir}(\theta, z) = \pi_\zeta^{\rm ud}(\theta, z + z_{\rm tilt})$

Ball, et al. *PPCF* (in prep).

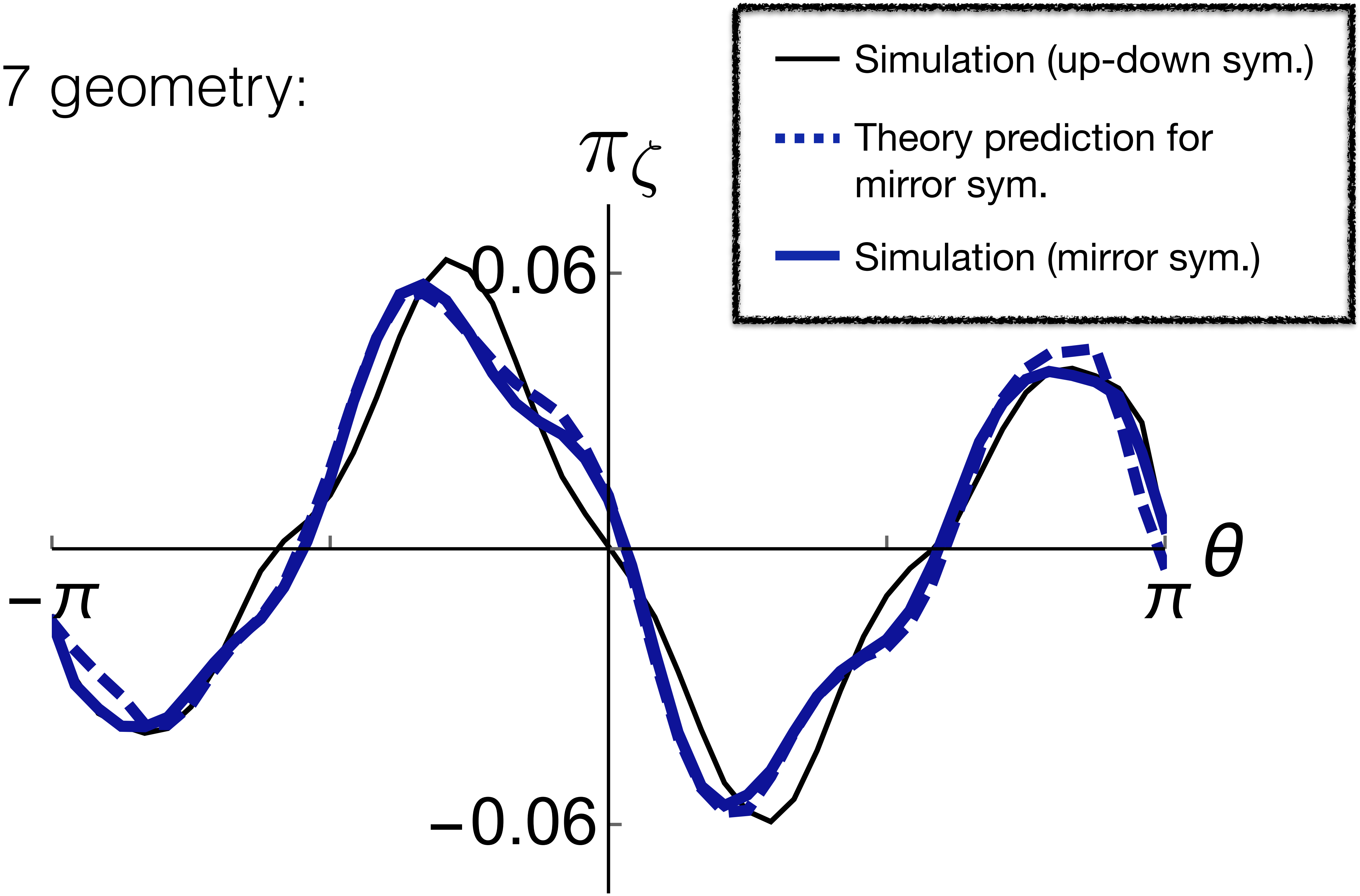
m=7 geometry:



Mom. flux distribution, testing $\pi_\zeta^{\mathrm{mir}}(\theta, z) = \pi_\zeta^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$

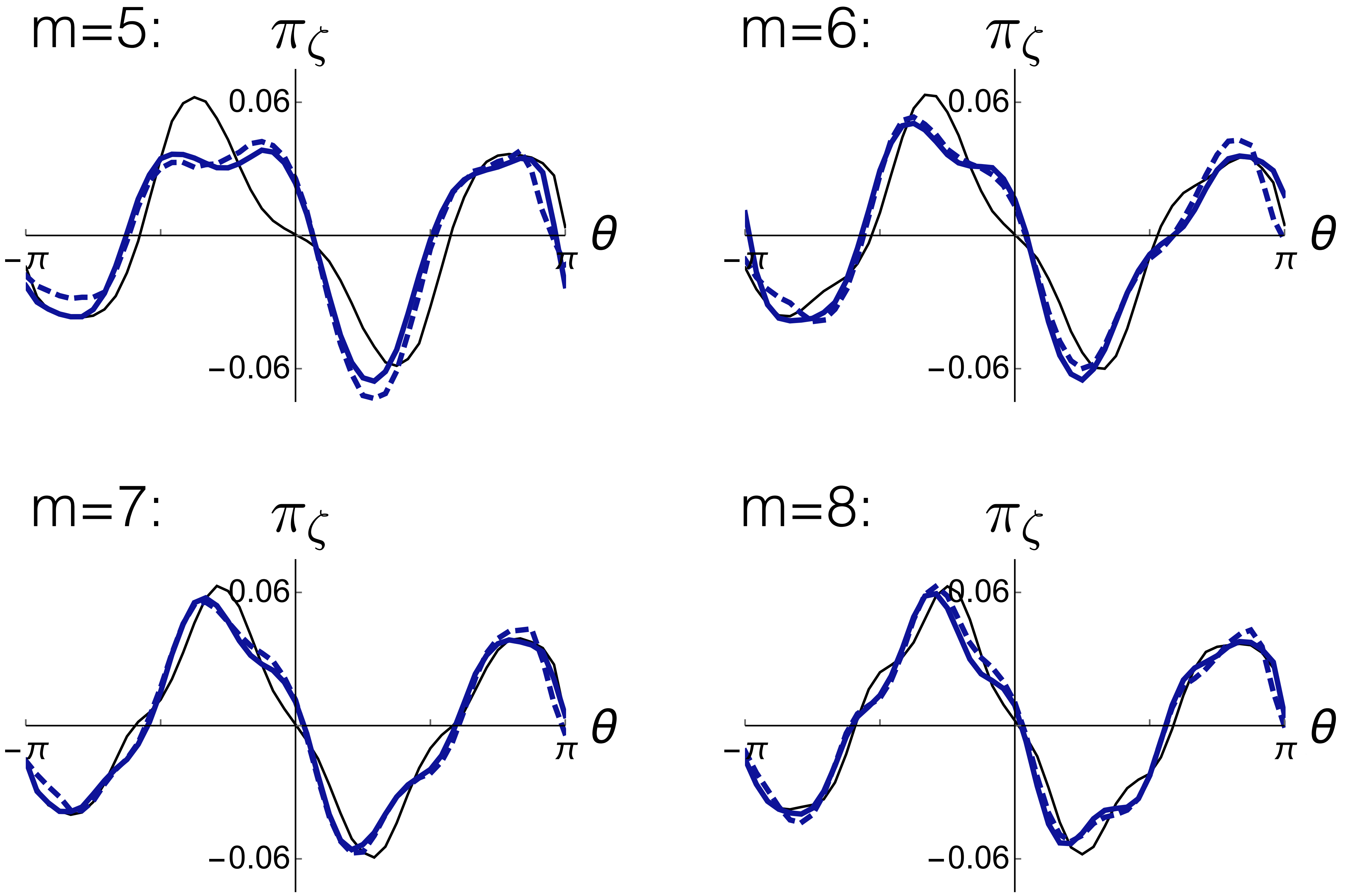
Ball, et al. *PPCF* (in prep).

m=7 geometry:



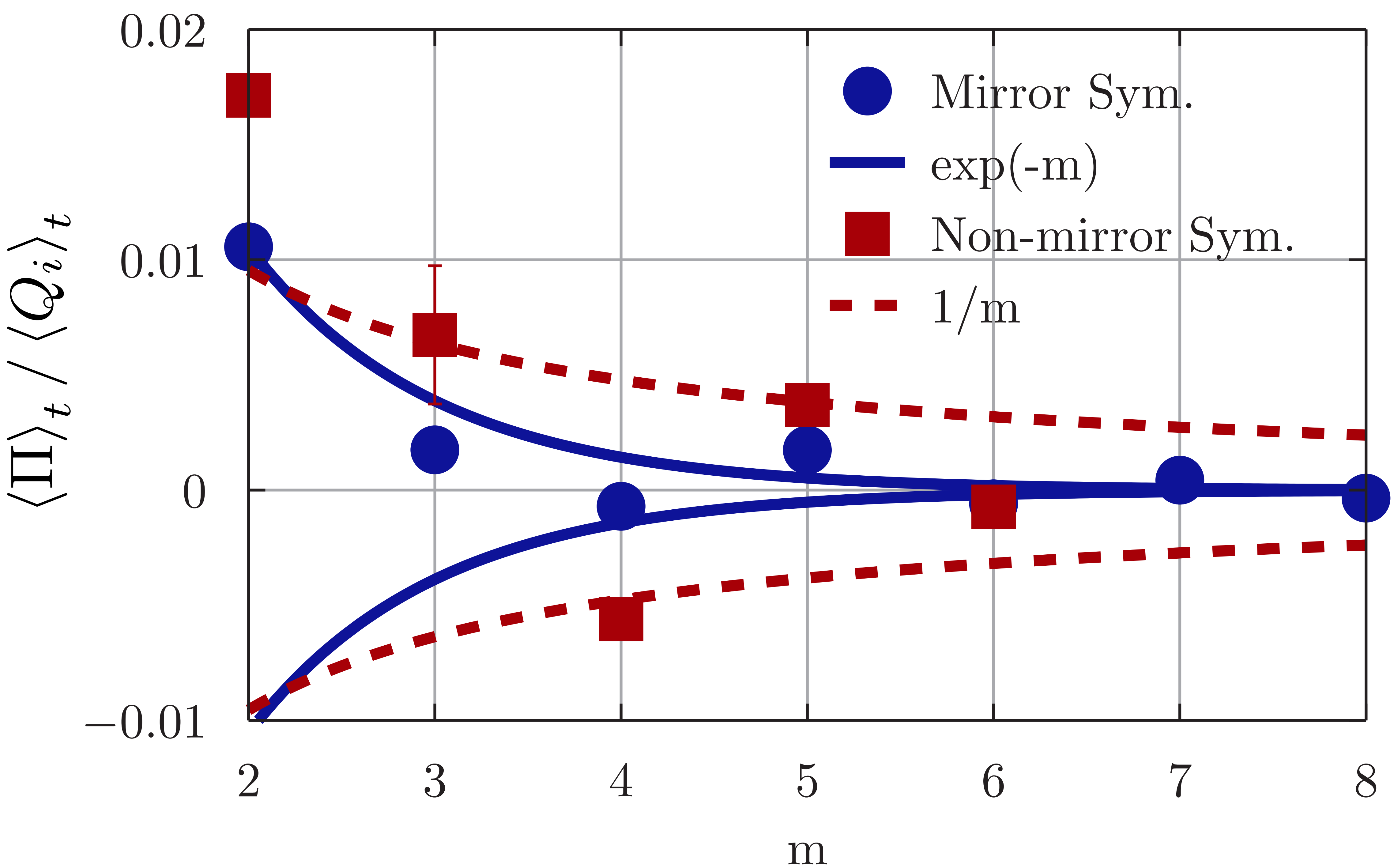
Mom. flux distribution, testing $\pi_\zeta^{\mathrm{mir}}(\theta, z) = \pi_\zeta^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$

Ball, et al. *PPCF* (in prep).



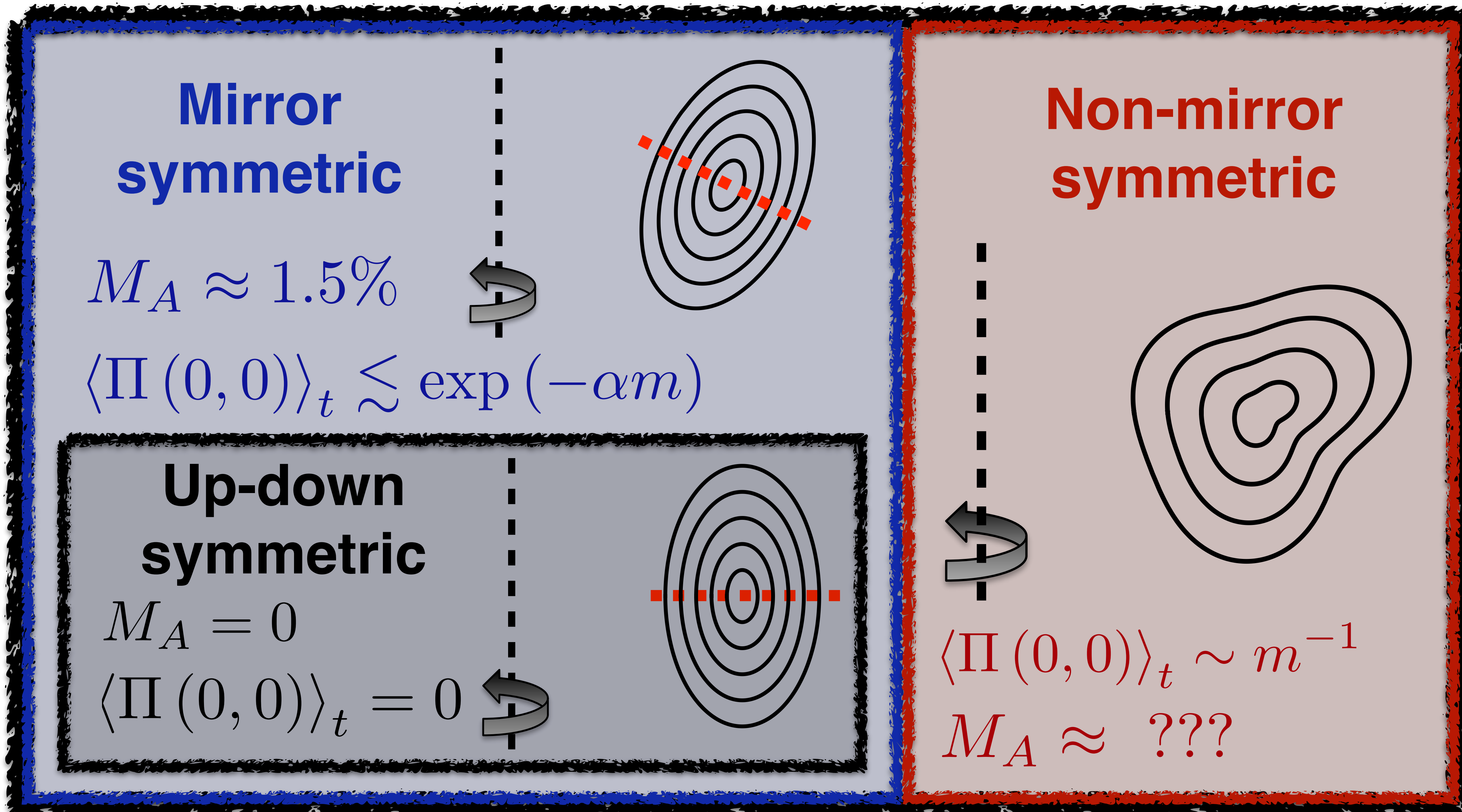
Numerical scaling with $m \gg 1$

Ball, et al. *PPCF* (in prep).



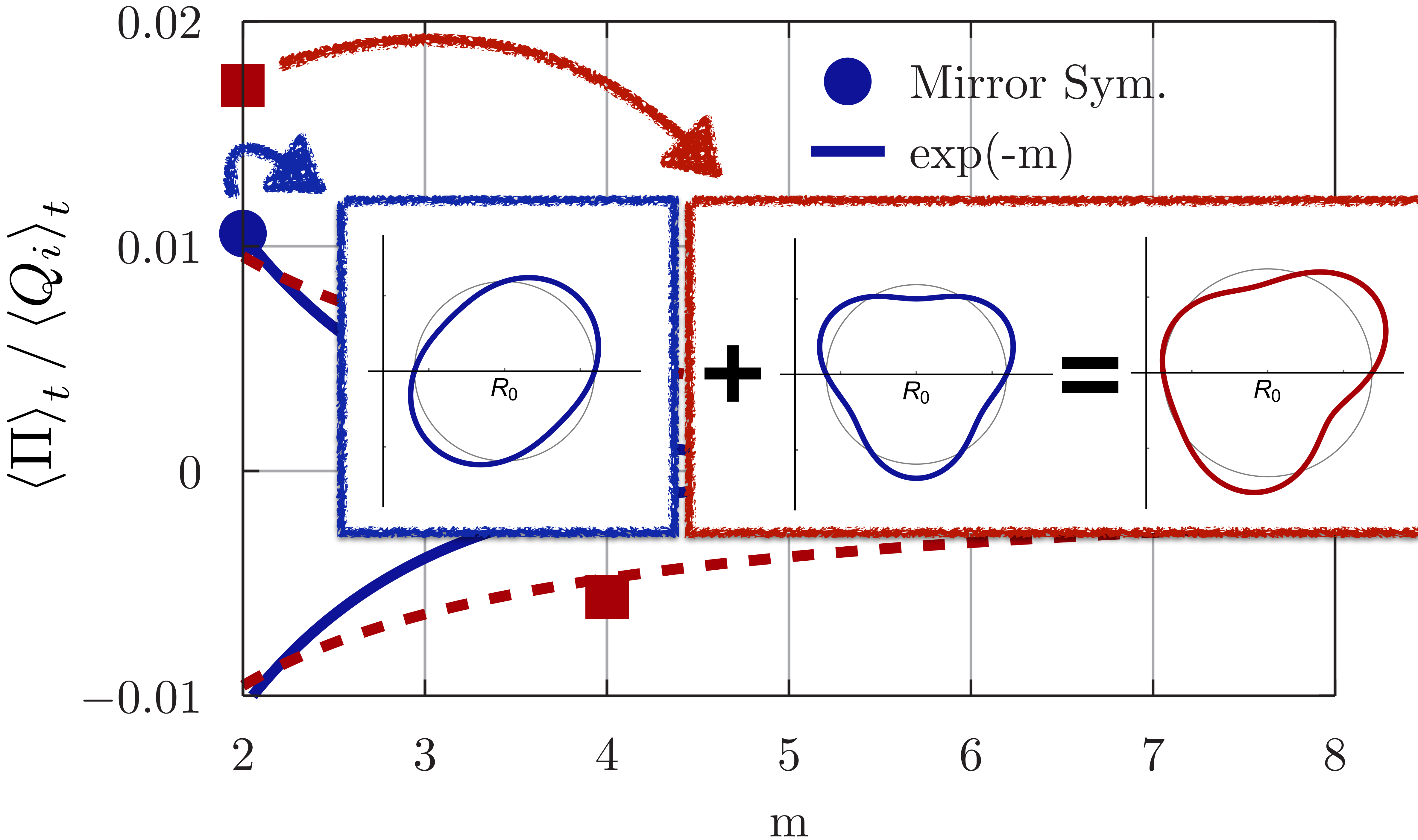
Outline

Tokamaks



Numerical scaling with $m \gg 1$

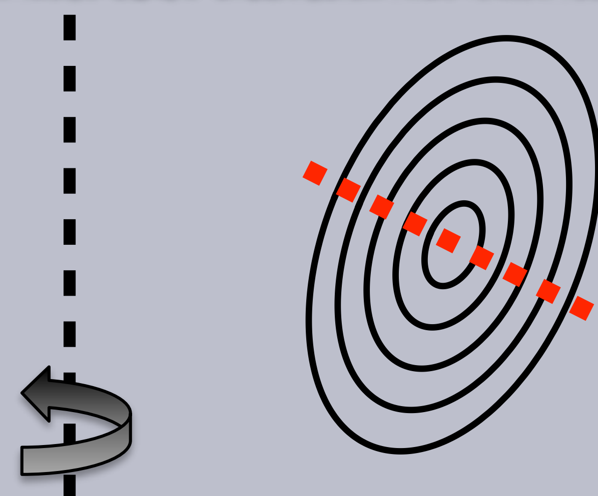
Ball, et al. *PPCF* (in prep).



Outline

Tokamaks

Mirror symmetric



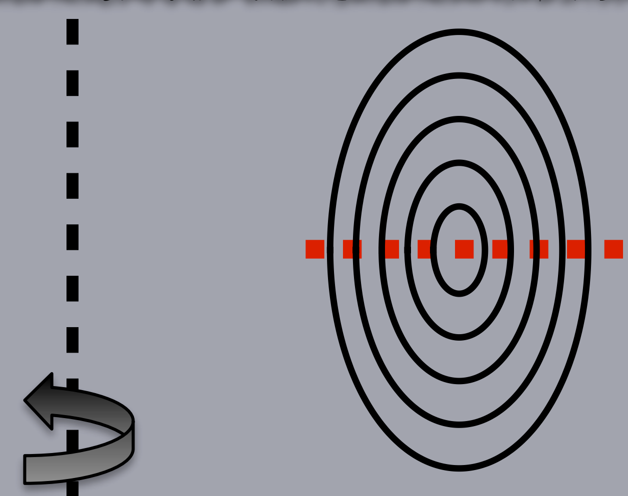
$M_A \approx 1.5\%$

$\langle \Pi(0,0) \rangle_t \lesssim \exp(-\alpha m)$

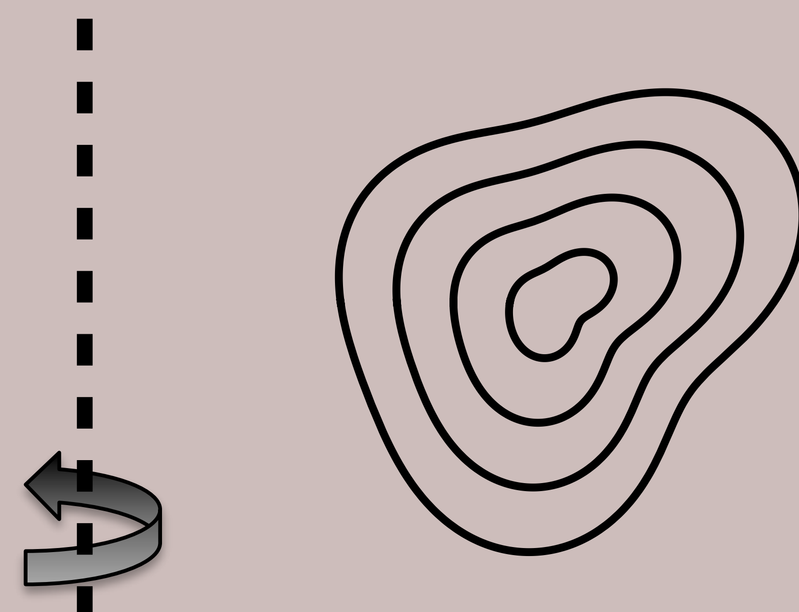
Up-down symmetric

$M_A = 0$

$\langle \Pi(0,0) \rangle_t = 0$



Non-mirror symmetric

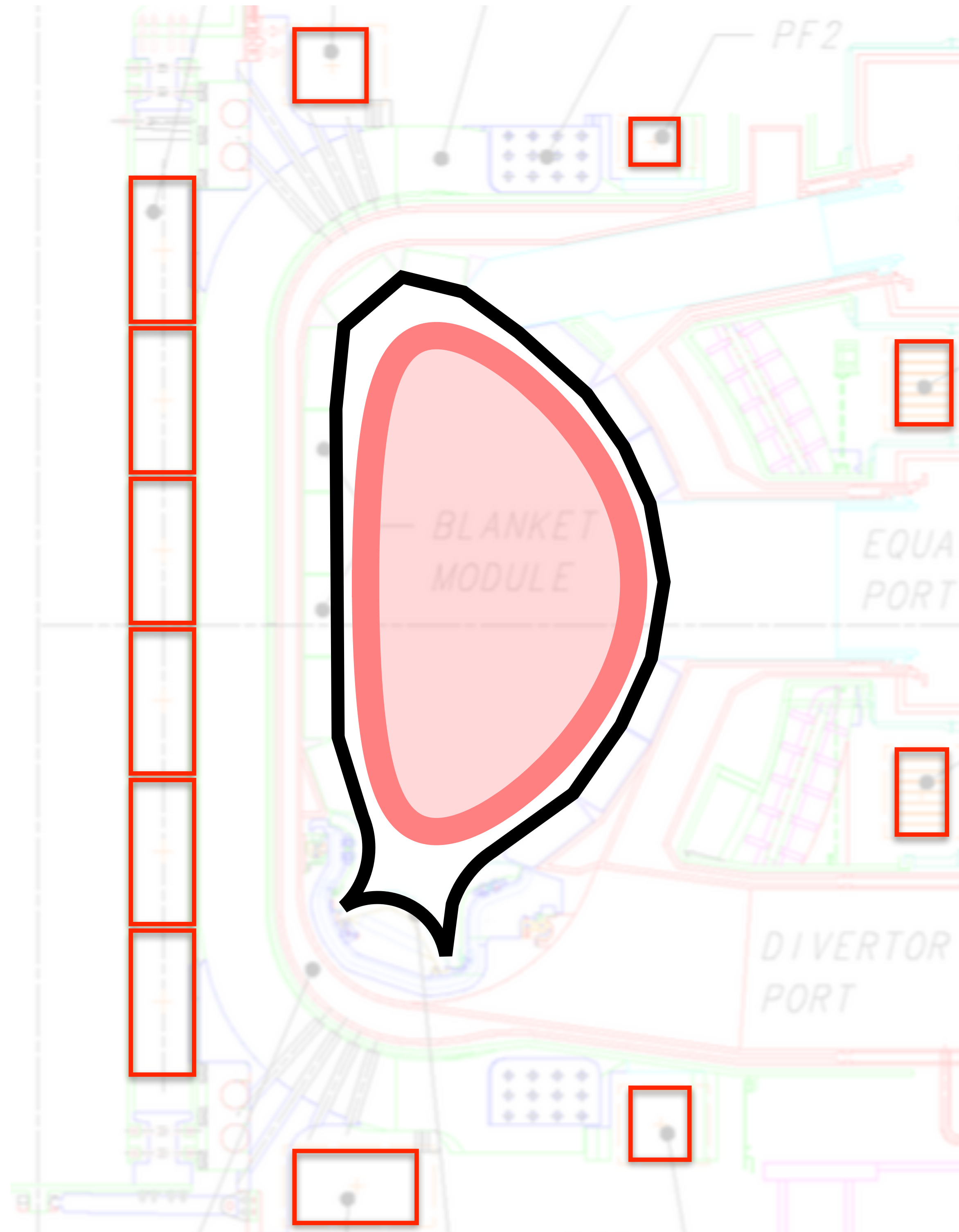


$\langle \Pi(0,0) \rangle_t \sim m^{-1}$

$M_A \approx 2.5\%?$

Can something like this be done in ITER?

- I think so, but there is a catch
- The shape of the first wall is fixed
- Each shaping coils has a current limit



Can something like this be done in ITER?

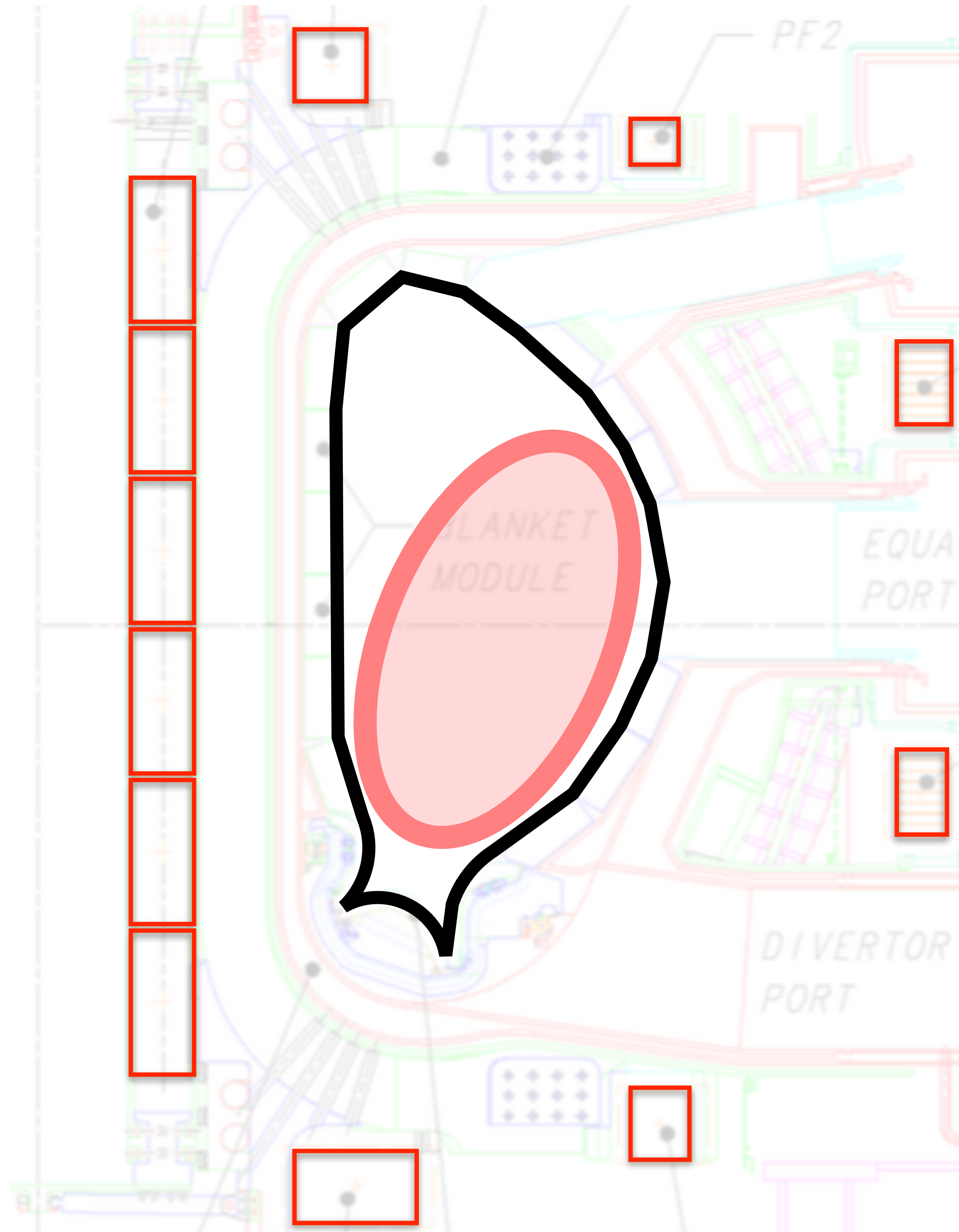
- I think so, but there is a catch
- The shape of the first wall is fixed

→ Reduced plasma volume

- Each shaping coils has a current limit

→ Reduced plasma current

- For $\beta_N = 3$, it's worth a shot



Conclusions

- Intrinsic rotation generated by up-down asymmetry scales well to larger machines (ITER, DEMO, etc.), unlike other mechanisms

- Tilting the elongation of flux surfaces by ~20° is a simple way to generate significant rotation

    - The magnitude of rotation is roughly what is needed to allow $\beta_N = 3$ in ITER

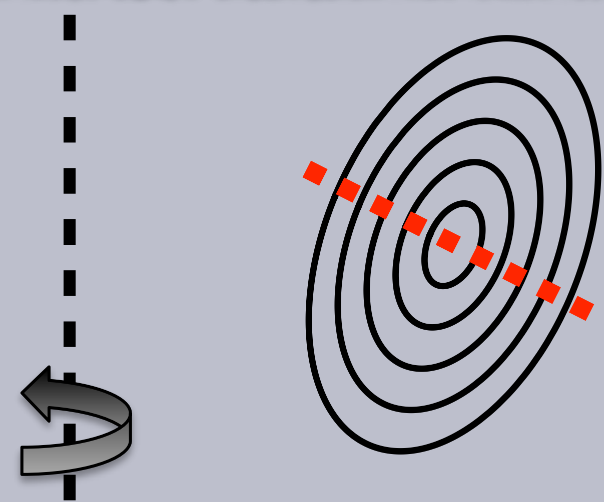
- Breaking mirror symmetry, in addition to up-down symmetry, has the potential to create even more rotation

Thank you!

Summary

Tokamaks

Mirror symmetric



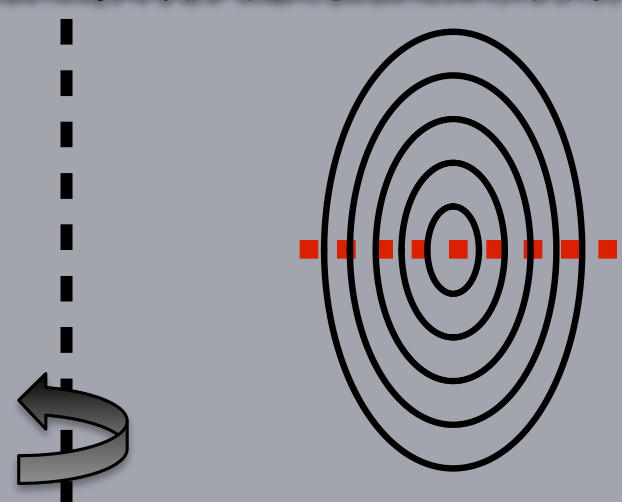
$M_A \approx 1.5\%$

$\langle \Pi \left(0,0\right) \rangle_t \lesssim \exp\left(-\alpha m\right)$

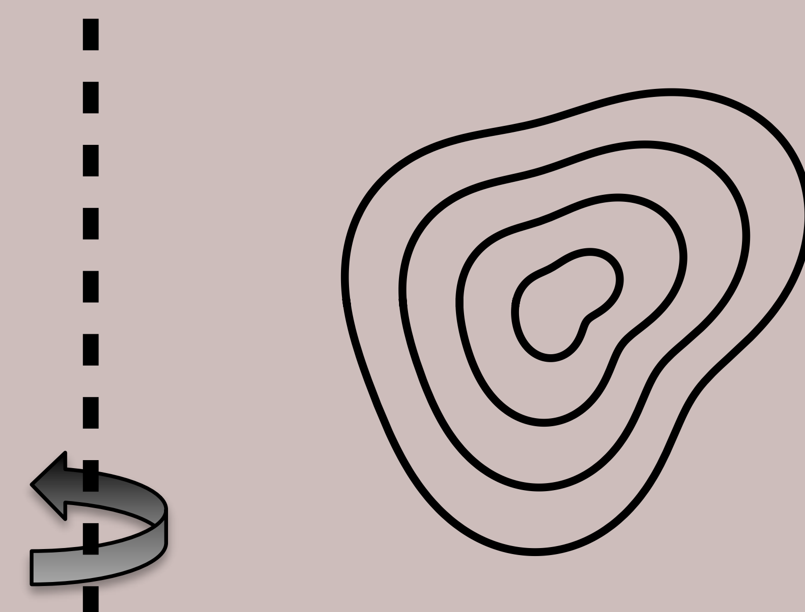
Up-down symmetric

$M_A = 0$

$\langle \Pi \left(0,0\right) \rangle_t = 0$



Non-mirror symmetric



$\langle \Pi \left(0,0\right) \rangle_t \sim m^{-1}$

$M_A \approx 2.5\%?$