Classical Coulomb Clusters in Harmonic Traps: Analytical Model

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Configurations of charged particles forming Coulomb clusters in potential traps have attracted the attention from various viewpoints. The traps are usually assumed harmonic and cylindrically symmetric, and the confining forces in axis and radial directions can be different. So, the confining potential for a particle of mass *m* is $U_t(r,z) = \frac{1}{2} m\omega^2 (r^2 + \alpha z^2)$, where ω is the frequency of oscillations in the *x*–*y* plain and α is the anisotropy parameter. One of the interesting problems is the restructuring of the cluster configuration when the trap anisotropy is changed. We present a simple model for evaluations of the cluster energy, size and shape depending of the trap anisotropy and the number of particles *N*.

Firstly, assuming homogeneous distribution of particles in cluster, we have obtained analytical expressions for the total potential energy *U* of the cluster, its radius *a* in the *x*–*y* plain and semiaxis *b* in the *z* direction. The cluster form is a spheroid, which is prolate if $\alpha < 1$ and oblate if $\alpha > 1$, for $\alpha = 1$ it is spherical.

Then we have evaluated influence of a possible inhomogeneity. If we propose simple inhomogeneous particle distribution $\rho(t) = \rho_0(1 - t^n)$, where $t = r^2/a^2 + z^2/b^2$, $0 \le t \le 1$ (spheroid consists of uniform layers of similar surface), we can make evaluation analytically. We have shown that the cluster shape is invariable, for $n = 1$ its linear size is $(5/3)^{1/3} = 1.186$ times larger than that for homogeneous cluster, while the energy differs by the factor 1.004. For $n > 1$ the differences are lover. Indeed, the particle distribution approaches to the homogeneous one as $n \to \infty$.

Next, we consider 2D clusters $(\alpha \to \infty)$ performing the limiting transition. However, in this case the surface particle density is not constant. Then we have calculated the 2D cluster with homogeneous surface density and shown that the inhomogeneity of 2D cluster has little effect on its potential energy as well in the case of 3D cluster.

It is obviously, the assumption of homogeneous cluster is valid only for rather large number of particles (apparently, $N > 10^3$). In order to extend our model to smaller clusters, we introduce to the equation for energy a correction factor based on the fitting of the Monte Carlo data for 2D clusters with different *N* and assume that it can be used for 3D clusters too. Indeed, the comparison of the model results for 3D clusters with $N = 10^1 - 10^2$ shows a reasonable agreement with molecular dynamical calculations.