CHARGED PARTICLES IN THE HMF MODEL

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1.. DYNAMIC CHARACTERIZATION OF THE SYSTEM

We consider a system of N particles interacting in a ring of radius one. The system is composed of two species of particles: N_1 particles of species 1 and $N_2 = N - N_1$ of species 2. The interaction potential is that of the Hamiltonian Mean Field (HMF) model and the nature of the interaction is given by the combination of particle species, much like in a charged particles model. We compute the equilibrium condition for the system and, through this, determine the geometry of phase space. We present molecular dynamics simulations to prove our findings. Moreover, we show that our model presents a "bicluster"formation for low energies, similar to the findings for the repulsive HMF model.

The Hamiltonian describing the system is given by

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} - \frac{1}{2N} \sum_{i,j=1}^{N} e_i e_j [1 - \cos(\theta_i - \theta_j)], \tag{1}$$

where

$$e_i = \begin{cases} +1 & \text{se } 1 \leq i \leq N_1 \\ -1 & \text{se } N_1 < i \leq N, \end{cases}$$

$$m_i = \begin{cases} M & \text{se } 1 \leq i \leq N_1 \\ m & \text{se } N_1 < i \leq N, \end{cases}$$

and the conjugated pair (θ_i, p_i) represents, respectively the position of the i-th particle on the ring $S_{2\pi}$ and its momentum. We assume there are two species of particles in the system: N_1 particles of charge $e_i = 1$ and $N_2 = N - N_1$ particles of charge $e_i = -1$. Therefore, the equations of motion for the i-th particle are

$$\begin{aligned} \dot{\theta}_i &= \frac{p_i}{m_i} \\ \dot{p}_i &= \frac{1}{N} \sum_j e_i e_j \sin(\theta_i - \theta_j) \end{aligned} \tag{2}$$

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We define the mean field quantities (we also use the term magnetization)

$$\mathbf{M}_1 = \left(\frac{1}{N_1}\sum_{i}^{N_1}\cos\theta_i, \frac{1}{N_1}\sum_{i}^{N_1}\sin\theta_i\right) \tag{3}$$

$$\mathbf{M}_2 = \left(\frac{1}{N_2}\sum_{i=N_1+1}^{N_2}\cos\theta_i, \frac{1}{N_2}\sum_{i=N_1+1}^{N_2}\sin\theta_i\right), \tag{4}$$

$$\mathbf{A} = c_1\mathbf{M}_1 - c_2\mathbf{M}_2, \tag{5}$$

with which we obtain

$$\dot{p}_i = e_i\left(\sin\theta_i A_x - \cos\theta_i A_y\right) \tag{6}$$

and

$$H = \sum_{i=1}^{N}\frac{p_i^2}{2m_i} - \frac{1}{2}\frac{(N_1-N_2)^2}{N} + \frac{N}{2}\left(A_x^2 + A_y^2\right). \tag{7}$$

We see, thus, that vector $\mathbf{A}$ behaves as a mean field acting on particles.

Equilibrium properties are determined using a maximization of entropy method assuming all particles to be statistically uncorrelated.

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