Isochronous Island Chains Multiplicity For Wave-Particle Interactions

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The Poincaré-Birkhoff Fixed Point Theorem states that a (r, s) resonance presents mr islands in phase space [1-3], where m indicates the number of chains and r is the number of islands in each chain. Although the theorem does not make any claim on the value of m [1-3], we generally find in the literature twist systems that present just one island chain [1]. In this paper, we consider a twist system that does not belong to this common case of single chains, and we show that it is possible to determine m by analyzing the resonant terms that act on the system. The considered system describes a relativistic charged particle moving in a uniform magnetic field and perturbed by a standing electrostatic wave. When the wave is given as a series of periodic pulses, we observe that it presents an infinite number of perturbative terms acting on the system. Many of these terms have the same winding number and they may generate islands in the same region of phase space. The superposition of resonant terms with the same winding number makes the number of chains vary as a function of the parameters of the wave [4]. For all the resonances, the number of chains is related to the amplitudes of the various resonant terms and, overall, the number of chains in phase space increases without limit as the wave period and wave number increase. For sufficiently large values of these parameters, all the resonances present more than one chain. Expanding the Hamiltonian and the map of the system to first order, we determine the position of the resonant islands in phase space. From these expressions, we obtain numerically the parameter space representing the number of chains as a function of the wave period and wave number. For the most important resonances, we also calculate analytically the parameter region that corresponds to a given number of island chains [4].

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